PERFORMANCE COMPARISONS OF ADAPTIVE ALGORITHMS FOR BLIND EQUALIZATION

André F. C. Vliese, Sergio L. Netto, and Paulo S. R. Diniz

Programa de Engenharia Elétrica/COPPE, DEL/EE
Universidade Federal do Rio de Janeiro
PO Box 68504, Rio de Janeiro, RJ, 21945-970, Brazil

ABSTRACT

In this paper, we compare several algorithms for blind channel equalization. The analysis includes the joint order detection and blind channel estimation by least squares smoothing (J-LSS), the adaptive version of the J-LSS algorithm, and the prediction error algorithm. Analysis are performed with respect to the computational complexity and convergence speeds of the algorithms.

1. INTRODUCTION

A very common problem in communications systems consists on the degradation of the transmitted signal by a nonideal channel. An ideal channel is the one that the signal at the receiver is a scaled and delayed version of the transmitted signal. That is, in an ideal channel, if $s(t)$ is the transmitted signal, then the received signal should be $\alpha s(t-\tau)$, where α is a gain factor and τ is the overall propagation delay. However, due to channel imperfections, elements such as additive noise and multichannel propagation cause signal interference and end up degrading the received signal. In order to solve this problem, we can employ special filters, commonly referred to as equalizers, whose function is to attenuate the distortions introduced by the channel, thus regenerating the original signal.

We may perform channel equalization using two different schemes, namely, channel identification, as depicted in Figure 1a, and inverse modeling, as depicted in Figure 1b. In the channel identification configuration, equalization is performed in two steps: first, the channel model is determined in parallel with the channel operation; then, in a second stage, the inverse of the model is cascaded to the channel operation during the transmission process. In the inverse modeling configuration, the model is directly placed in series with the channel.

In general, both equalization schemes are based on adaptive FIR filters, because adaptive IIR structures tend to present several convergence problems [1]. Typically, adaptive equalizers used in digital communication systems require an initial training period, during which a standard sequence is transmitted. After this training stage, the equalizer is switched to the direct mode (in series with the channel), and data transmission can be performed. There are, however, several cases where the training period is undesirable [2], such as multiuser and/or mobile systems.

This work was partially supported by CNPq and CAPES.

Due to these problems, in the past few years, several methods for channel equalization have been presented in the literature in an attempt to eliminate the training period. These methods are collectively known as blind equalization methods [3]–[4].

2. BLIND EQUALIZATION

Consider a linear, time-invariant discrete-time system with input $s(k)$, output $y(k)$, and impulse response $h(k)$. We assume that $s(k)$ is a white noise, that is, the samples of the input signal are independently and identically distributed, with zero mean and unit variance. The whole problem considered here is to regenerate $s(k)$, or equivalently to identify the inverse of $h(k)$, given the output sequence $y(k)$.

Consider then the blind equalization system depicted in Figure 2, where $n(k)$ is an additive noise, such that

$$y(k) = \sum_{m=0}^{\infty} h(k-m)s(m) + n(k) \tag{1}$$

Let $w(k)$ be the equalizer impulse response which is ideally related to the channel response $h(k)$ by

$$\sum_{k} w(k)h(l-k) = \delta(l) \tag{2}$$

If this is the case, then the equalizer is said to be ideal in the sense that it reconstructs the original sequence $s(k)$ [2]. In practice, $h(k)$ is unknown, and then equation (2) cannot be used to determine the equalizer coefficients. However, we can devise an iterative procedure to update the inverse impulse response $\hat{w}(i,k)$ (where the index i refers to the i-th coefficient at the time instant k). The iterative procedure is performed until the convolution between $\hat{w}(k)$ and the received signal $y(k)$ yields a complete (or at least partial) intersymbol interference removal. In such case, at the k-th

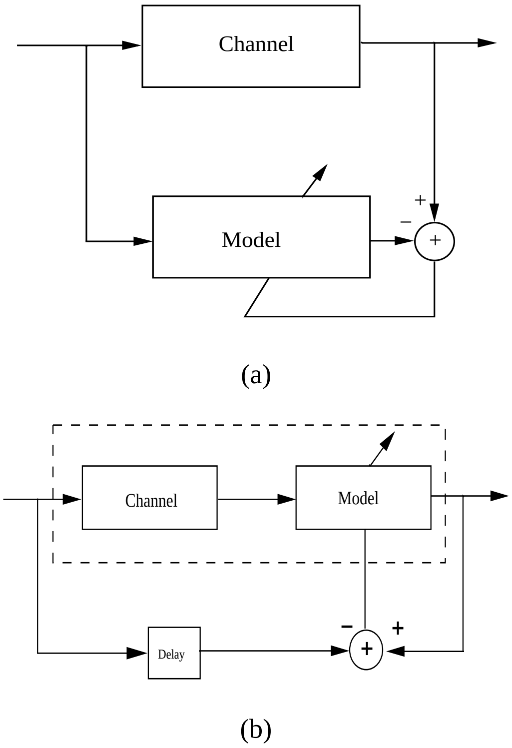

Figure 1: Block diagram of equalization schemes: (a) Channel identification; (b) Inverse modeling.

iteration, the approximated output sequence is given by

$$r(k) = \sum_{i=-L}^{L} \hat{w}(i,k)y(k-i) \quad (3)$$

We can then apply the output of the equalizer to a nonlinear system that generates an estimate $\hat{s}(k) = g[r(k)]$, and a estimate error $e(k) = \hat{s}(k) - r(k)$. If we attempt to minimize the mean squared value of this error we may employ an LMS-type adaptive algorithm [1], [2], by upgrading $\hat{w}(i,k)$ in the following manner:

$$\hat{w}(i,k+1) = \hat{w}(i,k) + \mu y(k-i)e(k) \quad (4)$$

where μ is the convergence factor of the algorithm. This equation describes the so-called bussgang algorithm for blind equalization, whose convergence can be easily analyzed [2].

In the following sections, we mention some blind equalization algorithms recently presented in the literature. We then perform several comparisons between

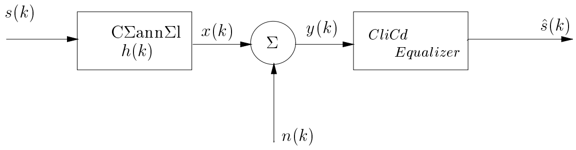

Figure 2: Model of a direct mode blind equalizer.

these algorithms with respect to convergence speed and computational complexity.

3. J-LSS ALGORITHM

The joint order detection and blind channel estimation algorithm by least squares smoothing (J-LSS) is characterized by the following steps [3]:

- It performs channel equalization using the system identification scheme depicted in Figure 1a.
- It is suitable for single-input and multiple-output (SIMO) channels.
- The smoothing technique that utilizes past and future samples of all signals involved.
- The algorithm is based on the concept that the projection error of the received signal on the input subspace must be equal to the projection error of the received signal on the output subspace.
- The order estimate is performed by trial and error, choosing the order value that minimizes the projection error of the received signal on the output subspace.

The J-LSS is fundamentally an off-line algorithm and its performance has been verified to be extremely dependent on the channel characteristics.

4. ADAPTIVE-LSS ALGORITHM

The adaptive version of the J-LSS (A-LSS) algorithm is a real-time adaptive algorithm for blind equalization characterized by [4]:

- It performs channel equalization using the system identification scheme depicted in Figure 1a.
- It is suitable for SIMO channels.
- It utilizes a data matrix with variable length using smoothing (past and future samples) technique.
- It is based on a lattice structure that performs error prediction in stages up to a point (order value) where such error is smaller than a given threshold.

- Channel modeling is then performed as in the J-LSS algorithm.

As the A-LSS is essentially an on-line version of the J-LSS algorithm, it tends to present a slightly decrease on the performance with respect to equalization ability.

5. PREDICTION ERROR ALGORITHM

The prediction error algorithm is characterized by [2]:

- It performs channel equalization using the inverse modeling scheme depicted in Figure 1b.
- It does not estimate the channel order, what adds robustness to its performance.
- It presents very low computational complexity.

It has been observed that adaptive algorithms for blind equalization tend to be very sensitive to channel noise.

6. COMPUTER EXPERIMENTS

In this section, we include a direct comparison of the distinct algorithms for blind equalization mentioned in the previous sections. We used $N = 500$ input samples and averaged over an ensemble of 100 experiments. We also considered several values of SNR at the input of the equalizer.

Experiment 1: In this case, we consider the two-output channel described by

$$h_1 = \begin{bmatrix} 0.354 & -0.715 \\ -0.016 & 0.690 \\ -0.324 & 0.625 \\ 0.209 & 0.120 \\ 0.253 & 0.388 \\ -0.213 & 0.132 \\ 0.254 & -0.120 \\ 0.118 & -0.388 \\ 0.483 & 0.451 \\ -0.034 & -0.204 \\ 0.462 & 0.560 \\ -0.111 & -0.675 \\ -0.285 & 0.147 \end{bmatrix} \tag{5}$$

whose zeros are depicted in Figure 3, corresponding to a well conditioned channel.

Performances of the adaptive equalizers (including a nonblind method based on the LMS algorithm) were measured with respect to the bit error rate (BER), for the input SNR ranging from 0 to 25 dB, in steps of 1 dB. The results are shown in Figure 4. From this figure, one can observe that nonblind methods present the better performance. While for blind methods, the J-LSS presented the best performance, very similar to the PE algorithm for the range $0 \leq \text{SNR} \leq 12$. Overall, The A-LSS algorithm presented the worst behavior among all methods considered here.

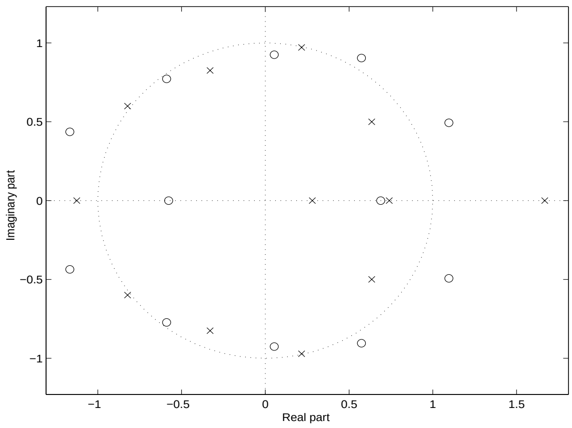

Figure 3: Zero plot of channel in Experiment 1.

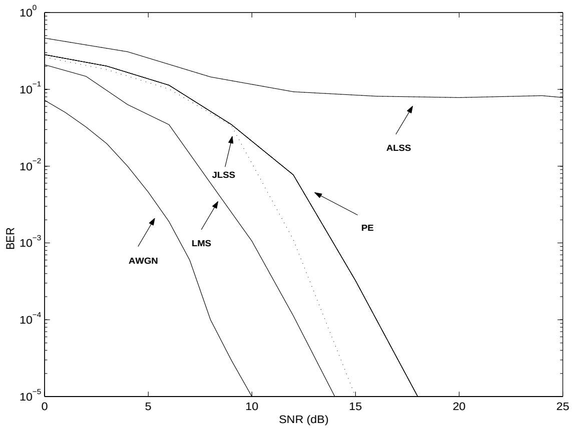

Figure 4: BER performance as a function of the input SNR for the equalization algorithms in Experiment 1.

The number of floating point operations required by each algorithm is summarized in Table 1.

Experiment 2: In this case, we consider the two-output channel described by

$$h_2 = \begin{bmatrix} 0.049 & 0.035 \\ -0.247 & -0.194 \\ 0.525 & 0.465 \\ -0.628 & -0.626 \\ 0.462 & 0.518 \\ -0.216 & -0.273 \\ 0.064 & 0.090 \\ -0.011 & -0.018 \\ 0.201 & 0.302 \\ -0.01 & -0.020 \end{bmatrix} \tag{6}$$

whose zeros are depicted in Figure 5. From this figure, one can clearly see the proximity of the zeros of the

Table 1: Computational complexity of equalization algorithms in Experiment 1.

Algorithm	flops
LMS	3.58×10^4
J-LSS	1.15×10^8
A-LSS	2.45×10^5
PE	1.08×10^5

Table 2: Computational complexity of equalization algorithms in Experiment 2.

Algorithm	flops
LMS	3.58×10^4
J-LSS	4.34×10^8
PE	3.45×10^5

two branches of the channel, what could cause some convergence problems for the adaptive algorithms.

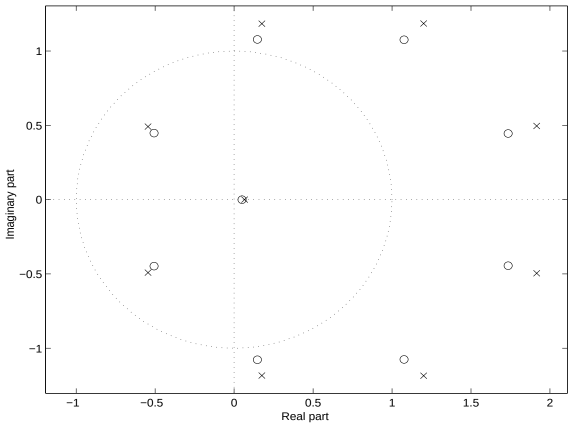

Figure 5: Zero plot of channel in Experiment 2.

Performances of the adaptive equalizers were measured with respect to the bit error rate (BER), for the input SNR ranging from 0 to 40 dB, in steps of 1 dB. The results are shown in Figure 6. From this figure, one can observe that nonblind methods present the better performance as expected once again. For then blind equalization methods, in this case the PE algorithm presented a slightly better performance.

The computational complexity of each algorithm in this case is given in Table 2.

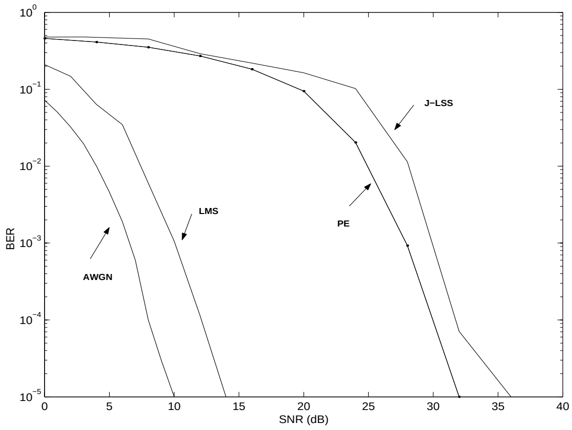

Figure 6: BER performance as a function of the input SNR for the equalization algorithms in Experiment 2.

7. CONCLUSION

The problem of adaptive blind equalization was considered. The performance of some adaptive algorithms for blind equalization was assessed. The analysis included the J-LSS, A-LSS, and PE algorithms recently proposed in the literature. It was observed that the J-LSS and PE algorithms presented somewhat similar results with respect to overall transmission rates. In addition, it was verified that the PE and A-LSS algorithms presented similar computational complexity.

8. REFERENCES

[1] P. S. R. Diniz, *Adaptive Filtering: Algorithms and Practical Implementation*, Kluwer Academic Publishers, Norwell, MA, 1997.

[2] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, 4th edition, Englewood Cliffs, NJ, 1998.

[3] L. Tong and Q. Zhao, “Joint order detection and blind channel estimation by least squares smoothing,” *IEEE Trans. Signal Processing*, vol. 47, no. 9, pp. 2345–2355, September 1999.

[4] Q. Zhao and L. Tong, “Adaptive blind channel estimation by least squares smoothing,” *IEEE Trans. Signal Processing*, vol. 47, no. 11, pp. 3000–3012, November 1999.