# AN ITERATIVE DESIGN OF FIR FILTERS WITH THE FREQUENCY-RESPONSE MASKING APPROACH

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## ABSTRACT

This paper presents an efficient method for designing FIR filters based on an iterative version of the frequency-response masking (FRM) approach. With the proposed method, the weight function for the FRM method is updated after each design in an attempt to generate an equiripple filter. The resulting procedure is then semi-automatic, as the margin gains of the FRM are estimated by the algorithm at each iteration, thus simplifying even more the overall filter design. The result is a very efficient filter in terms of the overall number of multiplications per output sample.

## 1. INTRODUCTION

The frequency-response masking (FRM) approach is a very efficient alternative for designing linear-phase FIR digital filters with large passbands and sharp transition bands. With such method, allowing an increase of the filter delay time, it is possible to reduce the filter complexity (number of multipliers and adders required per output sample) when compared to the standard design methods [1]. It has been verified that with the FRM approach without the concept of “don’t care” bands, the complexity reduction is to about 48% of the complexity yielded by the standard minimax approach. When using the concept of don’t care bands, the reduction increases even further to about 35% of the standard one. This results from the fact that in practice we can relax the specifications within the don’t care bands, and increase the weighting within the important bands of the filters required by the FRM method. In this paper, we present an iterative version for the FRM design, where the margin gains are updated at each iteration (partial design) in an attempt to generate an equiripple filter. The result is a simpler design method, as the optimal margin gains do not need to be estimated beforehand, and further reduction in the computational complexity of the final filter.

The organization of this paper is as follows: In Section 2, we describe the main concepts behind the FRM method. In Section 3 we then present an iterative extension of the FRM method and describe the whole procedure for designing a lowpass prototype FIR filter with reduced computational complexity. A design example with the proposed method is included in Section 4.

## 2. FREQUENCY-RESPONSE MASKING APPROACH

The basic block diagram for the FRM approach can be seen in Figure 1. In this scheme, the so-called interpolated base filter presents a repetitive frequency spectrum which is processed by the positive masking filter in the upper branch of this realization. Similarly, a complementary version of this repetitive frequency response is operated by the negative masking filter in the lower branch of the realization. In this procedure, both masking filters keep some of the spectrum repetitions which are then added together to compose the desired overall frequency response. The magnitude responses of the filter composing this sequence of operations are depicted in Figure 2, where one can clearly see the resulting filter with very sharp transition band.

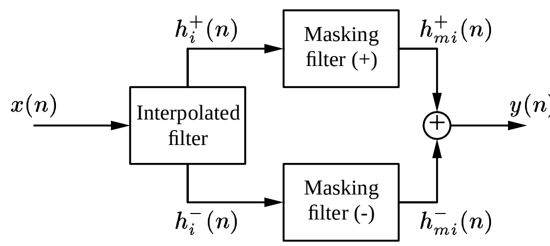
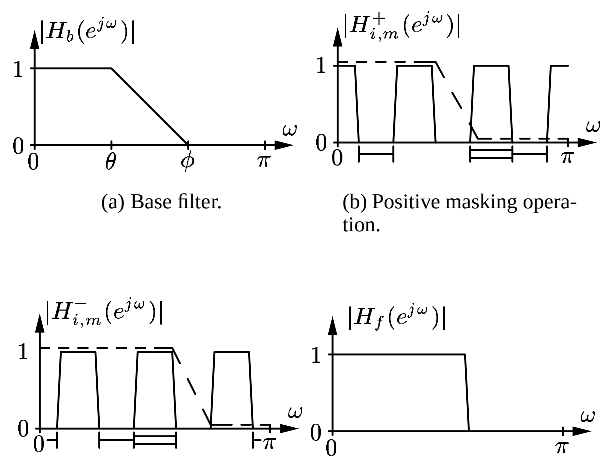



Figure 1: The basic realization of a reduced FIR filter using the FRM approach.




(a) Base filter.

(b) Positive masking operation.

(c) Negative masking operation.

(d) Resulting frequency-response.

Figure 2: Frequency-response masking approach, showing the don’t care bands (single line) and the critical bands (double lines below the frequency axis).

If the base filter has linear-phase and an even order $N$, its direct and complementary transfer functions are given by

$$H_i^+(z) = \sum_{i=0}^{N} h_b(i) z^{-Li} \quad (1)$$

$$H_i^-(z) = z^{-N/2} - \sum_{i=0}^{N} h_b(i) z^{-Li} \quad (2)$$

respectively, where $L$ is the interpolation factor and $h_b(n)$ is the impulse response of the base filter. From the equations above, we can readily see that

$$|H_i^-(e^{j\omega})| = 1 - |H_i^+(e^{j\omega})| \quad (3)$$

and also that $|H_i^-(e^{j\omega})|$ can be obtained by subtracting $|H_i^+(e^{j\omega})|$ from the signal at the central node in $H_i^+(z)$.

The cutoff frequencies $\theta$ and $\phi$ of the base filter (see Figure 2) depend on $L$ and on the desired band-edge frequencies $\omega_p$ and $\omega_s$ of the overall filter. The masking filters are simple FIR filters with band-edge frequencies that also depend on $L$ and on the bands of the interpolated filter. Therefore the optimal value of $L$ that minimizes the overall number of multiplications can be obtained by estimating the lengths of all sub-filters for various $L$ and finding the best case scenario empirically.

As the frequency responses in each branch depicted in Figure 2 are complementary, the corresponding passband ripples should cancel each other, specially if the two masking filters have approximately the same length. Using the concept of gain margins to determine the specifications for each sub-filter, we can see from the construction of the filter [1] that within the noncritical bands the overall ripple is approximately the sum of the ripple in one of the masking filters (depending on the frequency value) with a second-order term, due to the almost-perfect cancellation of the two branches. This fact must taken into consideration to determine the specifications for the passband ripple and the stopband attenuation in each subfilter of the FRM design.

For instance, in a design of a low-pass FIR with a desired bandpass ripple of 0.1 dB and minimum stop-band attenuation of 40 dB, the necessary worst-case margin at the noncritical bands is approximately 2.2%, while the worst-case margin at the critical bands are about 50% of the desired overall ripples. Therefore, the weighting at the noncritical bands should be relaxed and the weighting at the critical bands should be increased accordingly in order to accomplish the margin requirements in all frequency bands.

## 3. ITERATIVE FRM DESIGN

In this section, we describe the iterative version of the FRM design for improving the frequency response at the critical bands of the overall filter. We start the design by finding the appropriate values of the cutoff frequencies for the subfilters and the value of $L$ which will give the best reduction for the filter. One can then estimate the number of coefficients for the minimax approach, and verify the final response of the filter, reducing the number of coefficients if possible and redesigning the filter. After this, the masking filters can be designed, employing the concept of don't care bands, adjusting the weights in each band in such a way that the critical bands receive higher weights. The next step is to locate the repetition of the base filter spectrum which is responsible for the sharp transition of the filter. These frequencies are given by [1]

$$\omega_1 = m\frac{\pi}{L} \quad (4)$$

$$\omega_2 = (m+1)\frac{\pi}{L} \quad (5)$$

where $m$ is the largest integer such that $\omega_2$ is immediately below the largest cutoff frequency $\omega_s$ of the masking filters. These two frequencies, $\omega_1$ and $\omega_2$, are the centers of the first and second critical bands, respectively. Once these frequencies are determined, we can map the masking filter responses over the base filter response, and estimate the resulting error with

$$\begin{aligned} |H(e^{j\omega})| &= |H_m^+(e^{j\omega})H_i^+(e^{j\omega}) + H_m^-(e^{j\omega})H_i^-(e^{j\omega})| \\ &= |H_m^+(e^{j\omega})H_i^+(e^{j\omega}) + H_m^-(e^{j\omega})[1 - H_i^+(e^{j\omega})]| \end{aligned} \quad (6)$$

over the interval $\omega \in [\omega_1, \omega_2]$. As we are interested on optimizing the base filter, we can map the frequency responses of the masking filters back to the frequency interval $[0, \pi]$, yielding

$$|H(e^{j\omega})| = |H_m^+(e^{j\omega'})H_b(e^{j\omega}) + H_m^-(e^{j\omega'})[1 - H_b(e^{j\omega})]| \quad (7)$$

with, in this case, $0 \leq \omega \leq \pi$, and

$$\omega' = \omega_1 + (\omega_2 - \omega_1)\frac{\omega}{\pi} \quad (8)$$

if the positive masking filter has cutoff frequencies below the negative masking filter, or

$$\omega' = \omega_2 - (\omega_2 - \omega_1)\frac{\omega}{\pi} \quad (9)$$

if the positive masking filter has cutoff frequencies above the negative masking filter cutoff. This definition of $\omega'$ means that depending on which of the two branches is responsible for the last part of the passband, one needs to do a direct or inverse mapping on the frequency, according to equations (8) or (9), respectively. The last step is to determine the peak-constrained frequencies. For this project, we use the first bandstop peak ("side-lobe") of the masking filter. In the frequencies above this peak, it is supposed that the least-squares part of the base filter will cancel the other peaks of the masking filters. Thus, in each iteration, we seek for the first bandstop peak to determine where the envelope function is kept constant. Once the peak-constrained frequencies are known, the optimization algorithm can be applied to design the base filter. In Table 2 we see the design results for various frequencies specifications. Usually, the interpolation factor should be dependent of the sharpness of the transition band, but it can also be different for the two algorithms. In this case, the iterative FRM indicatesBy using the same value of $L$ in both algorithms, it is easier to compare the results, because the subfilters will keep the same frequency specifications for both algorithms, thus avoiding the masking filter to operate on different bands of the interpolated base filter.

The idea then is to analyze the resulting filter from the above procedure, determining the frequency intervals where the resulting gain deviates from the given specfications. For these frequency values we can then readjust the initial gain margins (increasing them in a direct proportion to the deviation) and perform a new FRM design. Such procedure can be repeated as many times is necessary to achieve an equiripple filter (if possible) or until the resulting filters do not improve in two consecutive iterations.

## 4. DESIGN EXAMPLE

As an example, we show the design of a lowpass reduced FIR filter, with cutoff frequencies of $\omega_p = 0.65\pi$ and $\omega_s = 0.66\pi$, maximum ripple at the passband of 0.2dB and minimum attenuation at the stopband of 40dB. The direct FIR filter implementation using a minimax design will require 382 coefficients, while using a standard FRM minimax with don't care bands the number of coefficients is reduced to 133 for the optimum choice of $L = 7$.

By using a WLS design [4] on the masking filters, we obtain the frequency-response depicted in Figure 5 (dashed lines). We can then notice from this figure, that by using $J = 5$ equirriple peaks in the WLS-Chebyshev design of the base filter we are able to restrict the critical peaks of the overall design. The overall filter amplitude response is shown in Figure 3, while in Figures 4 and 5 we see all the response at the critical bands.

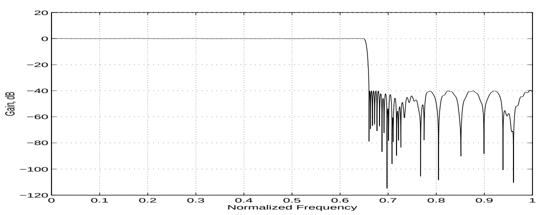



Figure 3: Amplitude response for the example filter.

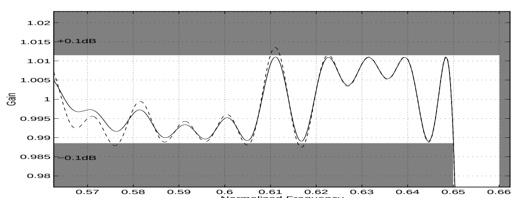



Figure 4: Amplitude response at the first critical band for the example filter (continuous line) and the response of the positive branch (dashed line).

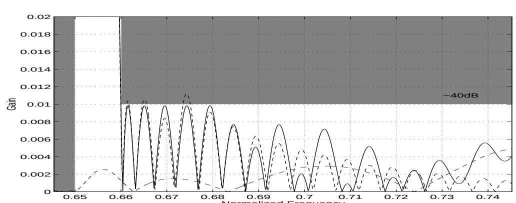



Figure 5: Amplitude response at the second critical band for the example filter (continuous line) and the responses of the two branches of the filter (dashed lines).

For this design, the result and the comparison between the minimax and the proposed approaches are shown in Table 1. In this table, $M$ denotes the number of coefficients for each of the sub-filters, $M_{Tot}$ is the total number of coefficients (multipliers) on the resulting filter, and the last column is the reduction factor, given by $M_{Tot}$ divided by the number of coefficients required by the direct implementation.

Table 1: Comparison between the designs using the minimax and the WLS-Chebyshev algorithms.

| Algorithm | $L$ | $M_b$ | $M_+$ | $M_-$ | $M_{Tot}$ | Red. Fact. |
|---|---|---|---|---|---|---|
| Minimax | 7 | 65 | 39 | 29 | 133 | 34.82% |
| WLS-Chebyshev | 7 | 57 | 32 | 26 | 115 | 30.1% |

In Table 2, we see the design results for various frequencies specifications. Usually, the interpolation factor should be dependent of the sharpness of the transition band, but it can also be different for the two algorithms. In this case, the current version of the iterative FRM performs only one additional iteration, when compared to the standard FRM design.

Table 2: Results obtained by using various frequency specifications. For these designs, the maximum allowable ripple at the passband is 0.2dB and the minimum attenuation is 40dB at the stopband.

| Specifications | | Minimax | | Iterative FRM | |
|---|---|---|---|---|---|
| $\omega_p$ | $\omega_s$ | $L$ | Red. Fact. | $L$ | Red. Fact. |
| $0.178\pi$ | $0.180\pi$ | 14 | 14.74% | 14 | 12.85% |
| $0.240\pi$ | $0.245\pi$ | 10 | 23.72% | 10 | 21.63% |
| $0.32\pi$ | $0.33\pi$ | 8 | 34.91% | 8 | 31.25% |
| $0.65\pi$ | $0.66\pi$ | 7 | 34.82% | 7 | 30.1% |

## 5. CONCLUSIONS

We introduced a new design method for FIR digital filters. The proposed method is an extension of the frequency-response masking (FRM) method in an iterative way, in an attempt to generate a resulting equiripple filter. The main advantages on the proposed algorithm are the flexibility to work with the weighting function, given any arbitrary set of specifications, and a resulting prototype filter which is very computationally efficient.

## 6. REFERENCES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