A homotopy continuation mapping for the Steiglitz-McBride adaptive algorithm

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ABSTRACT

In the last years, adaptive infinite-duration impulse response filters (IIR) have been studied as a possible alternative to adaptive finite-duration impulse response (FIR) filters. Some of the best known approaches for adaptive IIR filtering include the output error (OE), the equation error (EE), and the Steiglitz-McBride (SM) algorithms. In this paper, a homotopy continuation mapping (HCM) of the SM adaptive algorithm is proposed by using the OE and EE algorithms as basic schemes. The utilization of the homotopy factor results into a simple analysis of the general convergence behavior of the SM algorithm, allowing a direct comparison with the convergence properties of the OE and EE algorithms. Numerical examples are included in order to demonstrate the usefulness of the proposed mapping.

INTRODUCTION

In the last years, adaptive infinite-duration impulse response (IIR) filters have been studied as a possible alternative to adaptive finite-duration impulse response (FIR) filters. The main advantage of adaptive IIR filters when compared to adaptive FIR filters is their efficiency with respect to the number of coefficients when modeling systems with high selectivity poles. However, research has shown that adaptive IIR filters can present some serious implementation and convergence problems, such as: Possible existence of suboptimal (biased or local minimum) solutions, requirement of stability monitoring *etc.* To overcome these problems several techniques applicable to adaptive IIR filtering have been developed [1,2].

The two most commonly known approaches for adaptive IIR filtering are the output error (OE) and the equation error (EE) algorithms [1,2]. Probably the best feature of the OE and EE schemes is the fact that each algorithm can be associated to a performance surface [3]. This property allows a better understanding of the general convergence characteristics of these two adaptive techniques. The mean square output error (MSOE) performance surface associated to the OE algorithm has an unbiased global minimum, but also may present suboptimal local minima. At the same time, the EE is a simple adaptive algorithm that presents a unimodal mean square equation error (MSEE) performance surface and good stability characteristics. However, the EE algorithm may result in a biased solution in cases of presence of noise in the desired output signal.

Another algorithm for adaptive IIR filtering is the on-line version of the well known Steiglitz-McBride (SM) method for recursive system identification. In [4], Fan and Jenkins introduced a complete family of on-line versions of the SM method. Although the members of this group may present different transient properties, they are asymptotically equivalent in their steady-state characteristics. In fact this family of algorithms has convergence properties somewhere in between the OE and EE convergence behaviors. Basically, it has been proved in [5] that the SM family can correctly model an unknown system in cases of sufficient order identification, when the perturbation noise in the desired output signal is a white noise or nonexistent. In cases of insufficient modeling, the general behavior of the SM adaptive algorithms is somewhat unknown [6,7]. In those cases, it is known that this class of algorithms does not minimise the MSOE, although in some cases the final solution can be extremely close to the OE solutions [8].

In this paper, a homotopy continuation mapping (HCM) [9,10] of the SM adaptive algorithm is proposed by using the OE and EE algorithms as basic schemes. The utilization of the homotopy factor τ results into a sim-

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ple analysis of the general convergence behavior of the SM adaptive algorithm, allowing a direct comparison with the convergence properties of the OE and EE algorithms. This paper is organized as follows: In the next section, we introduce the OE, EE, and SM algorithms showing their respective characteristic equations. Later, the homotopy continuation mapping for the SM algorithm is presented. The following section, contains some system identification simulations showing how the proposed mapping can be used to analyse and interpret the convergence behavior of the SM algorithm.

PROBLEM STATEMENT

The general diagram of an adaptive IIR filter is shown in Figure 1. In this figure, $x(n)$ is the input signal, $y(n)$ is the desired output signal, $\hat{y}(n)$ is the adaptive output signal, and $e_{OE}(n)$ is the output error signal.

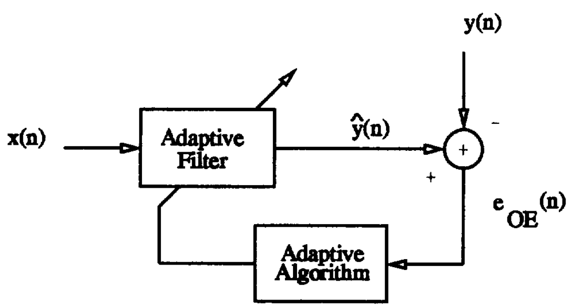

Figure 1: Block Diagram of a General Adaptive Filter

In a system identification problem, the desired output signal is assumed to be given by

$$y(n) = y_O(n) + v(n) \tag{1}$$

where $v(n)$ is the perturbation noise and $y_O(n)$ is the output of an unknown system or plant described by the transfer function

$$H(q) = \frac{B(q^{-1})}{A(q^{-1})} = \frac{b_0 + b_1 q^{-1} + \ldots + b_{n_b} q^{-n_b}}{1 - a_1 q^{-1} - \ldots - a_{n_a} q^{-n_a}} \tag{2}$$

where q^{-1} represents the unit delay operator, $B(q^{-1})$ and $A(q^{-1})$ are coprime polynomials, and the zeros of $z^{n_a} A(z^{-1})$ are assumed to be inside the unit circle. Similarly, $\hat{y}(n)$ is the output signal of the adaptive IIR filter characterized by

$$\begin{aligned} \hat{H}(q,n) &= \frac{\hat{B}(q^{-1},n)}{\hat{A}(q^{-1},n)} \\ &= \frac{\hat{b}_0(n) + \hat{b}_1(n) q^{-1} + \ldots + \hat{b}_{\hat{n}_b}(n) q^{-\hat{n}_b}}{1 - \hat{a}_1(n) q^{-1} - \ldots - \hat{a}_{\hat{n}_a}(n) q^{-\hat{n}_a}} \end{aligned} \tag{3}$$

By defining the adaptive filter coefficient vector as

$$\hat{\theta}(n) = \left[\hat{a}_1(n) \ldots \hat{a}_{\hat{n}_a}(n)\ \hat{b}_0(n) \ldots \hat{b}_{\hat{n}_b}(n)\right]^T \tag{4}$$

the basic form of a general adaptive algorithm can be written as

$$\hat{\theta}(n+1) = \hat{\theta}(n) + \mu(n) e(n) \hat{\phi}(n) \tag{5}$$

where $\mu(n)$ is a gain factor that can be a matrix or a scalar, $e(n)$ is an estimation error, and $\hat{\phi}(n)$ is the regressor or information vector associated to the adaptive algorithm. For the OE, EE, and SM adaptive algorithms, we have [1,2,4]:

i) Output Error (OE) Algorithm:

$$\begin{aligned} \hat{\phi}(n) &= [\hat{y}^f(n-1) \ldots \hat{y}^f(n-\hat{n}_a)\ x^f(n) \ldots x^f(n-\hat{n}_b)]^T \\ e(n) &= e_{OE}(n) = y(n) - \hat{y}(n) \end{aligned} \tag{6}$$

ii) Equation Error (EE) Algorithm:

$$\begin{aligned} \hat{\phi}(n) &= [y(n-1) \ldots y(n-\hat{n}_a)\ x(n) \ldots x(n-\hat{n}_b)]^T \\ e(n) &= e_{EE}(n) = \hat{A}(q^{-1},n) e_{OE}(n) \end{aligned} \tag{7}$$

iii) Steiglitz-McBride (SM) Algorithm:

$$\begin{aligned} \hat{\phi}(n) &= [y^f(n-1) \ldots y^f(n-\hat{n}_a)\ x^f(n) \ldots x^f(n-\hat{n}_b)]^T \\ e(n) &= e_{SM}(n) = \frac{\hat{A}(q^{-1},n)}{\hat{A}(q^{-1},n-1)} e_{OE}(n) \end{aligned} \tag{8}$$

where the superscript f in Equations (6) and (8) indicates that the respective signal is preprocessed by the all-pole filter $\frac{1}{\hat{A}(q^{-1},n)}$. In order to obtain a simpler form for the HCM, however, it is preferable to work with another member of the SM family of adaptive algorithms, namely the adaptive filter mode (AFM) [4], characterized by

$$\begin{aligned} \hat{\phi}(n) &= [y^f(n-1) \ldots y^f(n-\hat{n}_a)\ x^f(n) \ldots x^f(n-\hat{n}_b)]^T \\ e(n) &= e_{OE}(n) \end{aligned} \tag{9}$$

In the next section, we show the proposed mapping of the SM-AFM algorithm as a direct combination of the basic OE and EE adaptive schemes.

THE HOMOTOPY CONTINUATION MAPPING

A homotopy function is defined as [9,10]

$$\mathbf{h}(\hat{\boldsymbol{\theta}}, \tau) = \tau \mathbf{g}(\hat{\boldsymbol{\theta}}) + (1-\tau)\mathbf{f}(\hat{\boldsymbol{\theta}}) \tag{10}$$

where $\hat{\boldsymbol{\theta}}$ is the vector of parameters to be estimated and τ is the homotopy parameter, usually constrained to the interval $[0,1]$. The aim of a homotopy function is to find the solution of the system of equations $\mathbf{f}(\hat{\boldsymbol{\theta}}) = \mathbf{0}$

starting from the solution of another system of equations $\mathbf{g}(\hat{\boldsymbol{\theta}}) = \mathbf{0}$. In our case, we have

$$\mathbf{g}(\hat{\boldsymbol{\theta}}) = \frac{1}{2}\nabla_{\hat{\boldsymbol{\theta}}} E[e_{EE}^2(n)] = E[e_{EE}(n)\hat{\boldsymbol{\phi}}_{EE}(n)] \quad (11)$$

$$\mathbf{f}(\hat{\boldsymbol{\theta}}) = \frac{1}{2}\nabla_{\hat{\boldsymbol{\theta}}} E[e_{OE}^2(n)] = E[e_{OE}(n)\hat{\boldsymbol{\phi}}_{OE}(n)] \quad (12)$$

In order to force $\mathbf{h}(\hat{\boldsymbol{\theta}}, \tau)$ in Equation (10) to map the SM-AFM adaptive algorithm using the MSOE and MSEE formulations, τ should be modified to the form

$$\boldsymbol{\tau}(n) = diag[\tau_1(n) \ldots \tau_{\hat{n}_a}(n) \underbrace{0 \ldots 0}_{\hat{n}_b}] \quad (13)$$

where each $\tau_i(n)$, from Equations (6)-(12), will then be given by

$$\tau_i(n) = \frac{E[e_{OE}(n)y^f(n-i) - e_{OE}(n)\hat{y}^f(n-i)]}{E[e_{EE}(n)y(n-i) - e_{OE}(n)\hat{y}^f(n-i)]} \quad (14)$$

with $i = 1, \ldots, \hat{n}_a$.

Notice that the homotopy parameter given in Equation (13) is in fact a diagonal matrix, as each individual adaptive filter coefficient requires an independent HCM. The main reason for introducing the HCM for the SM adaptive algorithm is the possibility to analyze its convergence process based on the convergence behavior of the homotopy parameter. In fact when τ is close to one the SM algorithm is similar to the EE algorithm. Equivalently, as τ approaches zero the SM behavior becomes more and more alike the OE. In practice, it can be verified that the value of the homotopic parameter is not always constrained to the interval $[0, 1]$. From this fact, it can be inferred that the SM presents some singular characteristics different from the ones inherent to the basic OE and EE schemes.

The OE and EE adaptive algorithms approximate the gradients given in Equations (11)-(12) by their instantaneous values at each instant of time n. Consequently, an instantaneous approximation for the homotopy continuation factor would then be given by

$$\hat{\tau}_i(n) = \frac{e_{OE}(n)y^f(n-i) - e_{OE}(n)\hat{y}^f(n-i)}{e_{EE}(n)y(n-i) - e_{OE}(n)\hat{y}^f(n-i)} \quad (15)$$

Due to noisy characteristics of adaptive processes, however, this formula for the HCM tends to be unprecise and noisy. In the following section, we present some numerical examples and possible interesting analysis of the proposed HCM for the AFM-SM algorithm.

COMPUTATIONAL SIMULATIONS

Example I

In this example, let the plant be given by

$$H(q) = \frac{1}{1 - 0.8q^{-1}} \quad (16)$$

and the adaptive filter be characterized by $\hat{n}_a = 1$ and $\hat{n}_b = 0$. Let also the input signal $x(n)$ be a zero mean, unity variance Gaussian noise, $\mu = 0.001$, and $(\hat{a}_1(0); \hat{b}_0(0)) = (0; 0)$ be the initial point of the adaptive filter. Consider thus two distinct cases: A) The perturbation noise is a Gaussian noise $N[0, 0.1]$ independent to the input signal; B) The perturbation noise is the output of an IIR filter $V(q) = \frac{1}{1-0.6q^{-1}}$ to a Gaussian noise $N[0, 0.1]$ independent to the input signal. The convergence behavior for the homotopy parameter $\tau_1(n)$ for the two cases in this example can be visualized respectively in Figure 2 and Figure 3.

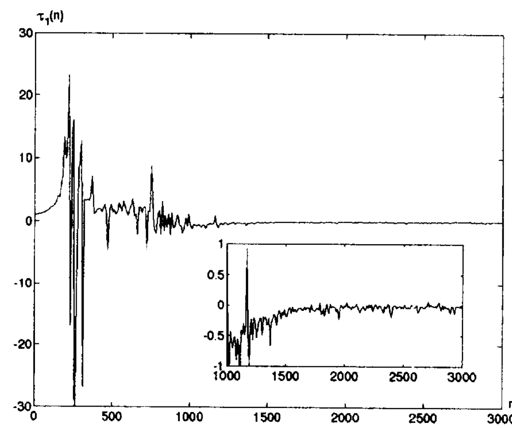

Figure 2: Convergence Behavior (and Detail) of the Homotopy Parameter: (A) White Noise Perturbation - Example I.

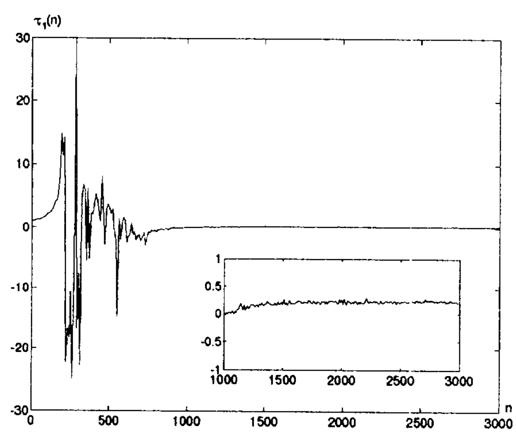

Figure 3: Convergence Behavior (and Detail) of the Homotopy Parameter: (B) Colored Noise Perturbation - Example I.

Both figures show that in the beginning of the adaptation process, the AFM-SM performance resembled the one of the EE algorithm, as $\tau_1(n) \approx 1$. Figure 2 (detail) also shows that in case (A), the homotopy parameter converges to zero, and consequently the SM algorithm tends to behave like the OE algorithm. From Figure 3 (detail), in the presence of colored noise (Case (B)), the SM algorithm becomes biased with respect to the MSOE solution, as the homotopic parameter does not converge to zero in average. In this example, we obtained $lim_{n\to\infty}\tau_1(n) \approx 0.237$.

Example II

In this example, let the plant be described by [7]

$$H(q) = \frac{0.05 - 0.4q^{-1}}{1 - 0.0003q^{-1} - 0.68915q^{-2}} \qquad (17)$$

and let the adaptive filter be of the same type as in Example I. Using the same initial conditions and μ of the previous example, the behavior of the homotopy parameter $\tau_1(n)$ in this case can be seen in Figure 4.

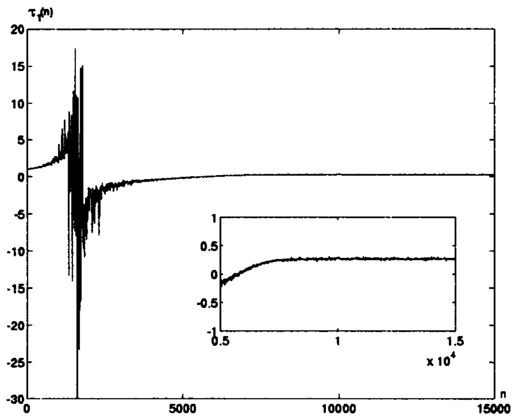

Figure 4: Convergence Behavior (and Detail) of the Homotopy Parameter - Example II.

From Figure 4, once again as $\tau_1(n) \approx 1$ in the beginning of the adaptive process, the SM algorithm resembles the EE algorithm. Also, after a somewhat intricate behavior at the intermediary part of the process, $\tau_1(n)$ approximates zero and the SM algorithm tends to behave similarly to the OE algorithm. Notice from Figure 4 (detail), however, that as $\tau_1(n)$ does not converge to zero in average, the SM algorithm cannot be said to minimize the MSOE in this example. In fact, in this case, we have the homotopic parameter converging to $lim_{n\to\infty}\tau_1(n) \approx 0.270$.

A final note about the experimental figures included in this paper must be added: Figures 2, 3 and 4 were obtained from an ensemble average of 1000 of the respective experiments above described. Also, in order to reduce the influence of the denominator of the fraction given in Equation (14) being close to zero in the final figures, a median filter [11] with window length equals 5 was used, and a subsequent decimation (by the order of 10) of the median filter output was performed.

CONCLUSIONS

In this paper, a homotopy continuation mapping for the SM adaptive algorithm was presented. The main motivation for this research is to associate the SM algorithm with two other adaptive algorithms, namely the OE and EE methods. By utilizing the homotopy continuation parameter, a clearer physical interpretation of the overall convergence behavior of the SM algorithm can be achieved.

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