A New Composite Adaptive IIR Algorithm

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Abstract

Adaptive infinite-duration impulse response (IIR) filters have been studied as a possible alternative to adaptive finite-duration impulse response (FIR) filters. Two of the best known approaches for adaptive IIR filtering include the output error (OE) and the equation error (EE) algorithms. In this paper, a new algorithm, the so-called composite square error (CSE) algorithm, for adaptive IIR filtering is introduced based on the explicit combination of the OE and and EE schemes. An innovative strategy for updating the composite factor and force the proposed algorithm to converge to the OE solution is also presented. Examples are included to demonstrate some of the interesting features of the new technique.

1 Introduction

In the last years, adaptive infinite-duration impulse response (IIR) filters have been studied as a possible alternative to adaptive finite-duration impulse response (FIR) filters. The main advantage of adaptive IIR filters when compared to adaptive FIR filters is their efficiency with respect to the number of coefficients when modeling systems with high selectivity poles. However, research has shown that adaptive IIR filters can present some serious implementation and convergence problems, such as: Possible existence of suboptimal (biased or local minimum) solutions, requirement of stability monitoring, slow convergence, *etc.* In an attempt to overcome these problems, several techniques applicable to adaptive IIR filtering have been presented in the literature [1,2].

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The two most commonly known approaches for adaptive IIR filtering are the output error (OE) and the equation error (EE) algorithms [1,2]. Probably the best feature of the OE and EE schemes is the fact that each algorithm can be associated to a performance surface [3]. This property allows a better understanding of the general convergence characteristics of these two adaptive techniques.

The mean square output error (MSOE) performance surface associated to the OE algorithm has an unbiased global minimum when the additional noise is independent to the input signal. However, this surface may present suboptimal local minima in cases of insufficient order modeling[1], or when the unimodality condition of Söderström [4] is not satisfied in cases of sufficient order identification. In cases of strictly sufficient order modeling, the unimodality of the MSOE performance surface can be guaranteed if the more general sufficient condition of Nayeri [5] is satisfied.

On the other hand, the EE is a simple adaptive algorithm that presents a unimodal mean square equation error (MSEE) performance surface and good stability characteristics. However, the EE algorithm may result in a biased solution, as in cases of presence of measurement/modeling noise in the desired output signal.

In this paper, we introduce the composite square error (CSE) algorithm that attempts to combine the good individual characteristics of both the OE and EE adaptive IIR algorithms. In order to allow a better control on the overall properties of the CSE algorithm, the composition of the OE and EE algorithms is made in a explicit form, following the approach of Kenney

[1]If the orders of both the numerator and denominator polynomials of the adaptive filter are greater or equal to those of the plant or unknown system, the case is called sufficient order identification. If the orders are both equal, we call it a strictly sufficient order case. Otherwise, it is called an insufficient order problem of system identification.

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and Rohrs in [6]. By composing the error square signal with the square values of the OE and EE signals, as opposed to the simple addition of these individual signals as in [7], an easier way to determine the performance surface associated to the resultant composite algorithm is achieved through the direct composition of the MSOE and MSEE error functions. This fact results into simpler analyses of the final convergence properties of the resultant algorithm.

This paper is organized as follows: In the next section, we present the OE and EE algorithms for adaptive IIR filtering showing their respective updating equations. Section III introduces the composite square error algorithm and shows its direct relationship with the OE and EE basic schemes. In Section IV, we present an innovative approach to update the composite parameter γ and force the new algorithm to converge to the optimal MSOE solution, n usually required feature for adaptive algorithms, specially in insufficient order identification cases. Section V contains some system identification simulations showing some interesting features of the newly proposed techniques.

2 Problem statement

The general diagram of an adaptive IIR filter is shown in Figure 1. In this figure, $x(n)$ is the input signal, $y(n)$ is the desired output signal, $\hat{y}(n)$ is the adaptive output signal, and $e_{OE}(n)$ is the output error signal. In a system identification problem, the desired output

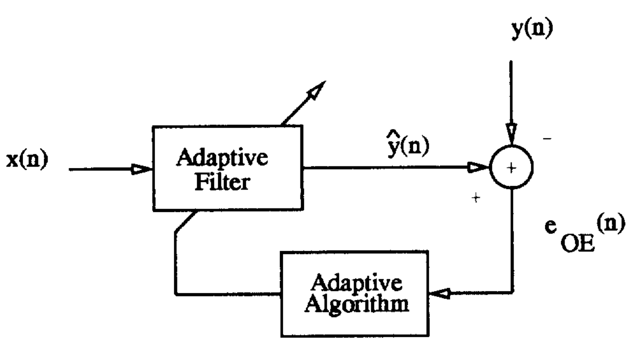

Figure 1: Block Diagram of a General Adaptive Filter

signal is assumed to be given by

$$y(n) = y_O(n) + v(n) \tag{1}$$

where $v(n)$ is the perturbation noise and $y_O(n)$ is the output of an unknown system or plant described by the transfer function

$$H(q) = \frac{B(q)}{A(q)} = \frac{b_0 + b_1 q^{-1} + \ldots + b_{n_b} q^{-n_b}}{1 - a_1 q^{-1} - \ldots - a_{n_a} q^{-n_a}} \tag{2}$$

where q^i represents the shift operator defined by $q^i[x(n)] = x(n-i)$, $q^{-n_b}B(q)$ and $q^{-n_a}A(q)$ are relatively coprime polynomials in q, and the zeros of $z^{n_a}A(z)$ are assumed to be inside the unit circle $|z|=1$; $z \in \mathcal{C}$. Similarly, $\hat{y}(n)$ is the output signal of the adaptive IIR filter characterized by

$$\hat{H}(q,n) = \frac{\hat{B}(q,n)}{\hat{A}(q,n)} = \frac{\hat{b}_0(n) + \hat{b}_1(n)q^{-1} + \ldots + \hat{b}_{n_{\hat{b}}}(n)q^{-n_{\hat{b}}}}{1 - \hat{a}_1(n)q^{-1} - \ldots - \hat{a}_{n_{\hat{a}}}(n)q^{-n_{\hat{a}}}} \tag{3}$$

By defining the adaptive filter coefficient vector as

$$\hat{\theta}(n) = \left[\hat{a}_1(n)\ldots\hat{a}_{n_{\hat{a}}}(n)\ \hat{b}_0(n)\ldots\hat{b}_{n_{\hat{b}}}(n)\right]^T \tag{4}$$

the basic form of a general adaptive algorithm can be written as

$$\hat{\theta}(n+1) = \hat{\theta}(n) + \mu(n)e(n)\hat{\phi}(n) \tag{5}$$

where $\mu(n)$ is a gain factor that can be a matrix or a scalar, $e(n)$ is an estimation error, and $\hat{\phi}(n)$ is the regressor or information vector associated to the respective adaptive algorithm.

The Output Error (OE) algorithm minimizes the mean-square output error defined as $E[e_{OE}^2(n)]$, and consequently for the OE algorithm we have

$$\hat{\phi}_{OE}(n) = [\hat{y}^f(n-1)\ldots\hat{y}^f(n-n_{\hat{a}})\ x^f(n)\ldots x^f(n-n_{\hat{b}})]^T$$
$$e_{OE}(n) = y(n) - \hat{y}(n) \tag{6}$$

where the superscript f in indicates that the respective signal is preprocessed by the all-pole filter $\frac{1}{\hat{A}(q^{-1},n)}$. Meantime, the Equation Error (EE) algorithm attempts to minimize the mean-square equation error given by $E[e_{EE}^2(n)]$ and the EE algorithm is characterized by

$$\hat{\phi}_{EE}(n) = [y(n-1)\ldots y(n-n_{\hat{a}})\ x(n)\ldots x(n-n_{\hat{b}})]^T$$
$$e_{EE}(n) = \hat{A}(q^{-1},n)e_{OE}(n) \tag{7}$$

3 The combined square error adaptive algorithm

Let us now introduce a new IIR adaptive algorithm, so-called the combined square error (CSE) algorithm, that explicitly combines the OE and EE algorithms in the following form

$$e_{CSE}^2(n) = \gamma e_{EE}^2(n) + (1-\gamma) e_{OE}^2(n) + K \qquad (8)$$

where γ is the combining parameter and $K \geq 0$ is a constant that guarantees the right-hand side of the above equation to be nonnegative for a general range of the values of γ. Notice that if K is set to zero, the combining parameter γ must be constrained to the interval $[0,1]$ in order to assure coherence and validity to the definition of $e_{CSE}^2(n)$ given above. Using equation (8), the mean combined square error (MCSE) performance surface associated to the CSE algorithm can be directly calculated as being described by

$$E[e_{CSE}^2(n)] = \gamma E[e_{EE}^2(n)] + (1-\gamma) E[e_{OE}^2(n)] + K \quad (9)$$

i.e., the MCSE performance surface is obtained by the weighted combination of the MSOE and MSEE surfaces plus a constant $K \geq 0$ that assures the nonnegativity of the MCSE function.

Obtaining the updating equation characteristic of the CSE adaptive algorithm based on a steepest descent minimization scheme, we have

$$\hat{\boldsymbol{\theta}}(n+1) = \hat{\boldsymbol{\theta}}(n) - \mu' \boldsymbol{\nabla}_{\hat{\boldsymbol{\theta}}}[e_{CSE}^2(n)] \qquad (10)$$

where, from equation (8), the composite square gradient vector $\boldsymbol{\mathcal{V}} = \boldsymbol{\nabla}_{\hat{\boldsymbol{\theta}}}[e_{CSE}^2(n)]$ is given by

$$\begin{aligned}\boldsymbol{\mathcal{V}} &= \gamma \boldsymbol{\nabla}_{\hat{\boldsymbol{\theta}}}[e_{EE}^2(n)] + (1-\gamma)\boldsymbol{\nabla}_{\hat{\boldsymbol{\theta}}}[e_{OE}^2(n)] \\ &= -2[\gamma e_{EE}(n)\hat{\boldsymbol{\phi}}_{EE}(n) + (1-\gamma) e_{OE}(n)\hat{\boldsymbol{\phi}}_{OE}(n)] \quad (11)\end{aligned}$$

This equation shows that the instantaneous gradient vector of the CSE algorithm is a combination of the instantaneous gradient vectors of the OE and EE adaptive algorithms, as expected due to the definition used for the combined square error signal.

4 A time-varying composite parameter for the CSE algorithm

As mentioned before, the EE algorithm possesses some interesting convergence properties as overall stability and unique solution. Unfortunately, however, the final solution for this algorithm tends to be biased in the presence of perturbation signal. On the other hand, the OE algorithm is characterized by possibly unstable adaptation and/or convergence to suboptimal solutions. However, the global optimum solution of the OE algorithm is proved unbiased even in the presence of any kind of perturbation signal statistically independent of the input signal. Consequently, one could conclude that an ideal kind of adaptive algorithm would be the one that combines the good initial features of the EE algorithm, as good stability properties and unique solution, with the good final property of the OE of unbiased global optimum solution. This can be achieved by using the proposed CSE algorithm with a time-varying composite parameter $\gamma \equiv \gamma(n)$ with value initially set to one and approximating zero as the adaptation process converges. One form to implement this approach would be to use a time-varying composite factor with a recursive updating equation of the form

$$\begin{aligned}\gamma(n+1) &= \gamma(n) - \mu_\gamma \boldsymbol{\nabla}_\gamma[e_{CSE}^2(n)] \\ &= \gamma(n) - \mu_\gamma[e_{EE}^2(n) - e_{OE}^2(n)] \qquad (12)\end{aligned}$$

and limiting $\gamma(n)$ to the interval $\gamma(n) \in [0,1]$ by applying a saturation procedure to the value of $\gamma(n)$ given by the above equation. Notice, however, that as the above scheme is based on the minimization of the mean composite square error, $\gamma(n)$ will converge to one if $E[e_{EE}^2(n)]^* < E[e_{OE}^2(n)]^*$, or zero in case of $E[e_{EE}^2(n)]^* > E[e_{OE}^2(n)]^*$, where $E[e_{EE}^2(n)]^*$ and $E[e_{OE}^2(n)]^*$ are respectively the minimum mean square equation error and output error values. In order to force the composite parameter to converge to zero in all cases, we need to modify equation (12) to the form

$$\gamma(n+1) = \gamma(n) - \mu_\gamma |e_{EE}^2(n) - e_{OE}^2(n)| \qquad (13)$$

and also continue to make sure that value of the composite parameter stays in the interval $\gamma(n) \in [0,1]$. As a final note for this section, it should be reemphasized that the need for constraining the value of $\gamma(n)$ to the closed interval $[0,1]$ is to guarantee the maintenance of a mathematical and physical meaning to the composite square error signal.

5 Computational simulations

In this example, consider the plant described by [8]

$$H(q) = \frac{0.05 - 0.4q^{-1}}{1 - 0.0003q^{-1} - 0.68915q^{-2}} \qquad (14)$$

and let the adaptive filter be characterized by $n_{\hat{a}} = 1$ and $n_{\hat{b}} = 0$. Assume also a zero mean, unitary variance Gaussian noise as input signal $x(n)$ and no perturbation noise being present in the desired output signal. Let us apply the proposed CSE algorithm with time-varying composite parameter to perform the insufficient order identification problem above described.

Assume $\mu = 0.002$, $\mu_\gamma = 0.0015$, $\gamma(0) = 1$, and zero initial conditions, *i.e.*, let $[\hat{a}_1(0)\ \hat{b}_0(0)] = [0\ 0]$ be the initial point for the adaptive filter.

Figure 2 depicts the convergence of the adaptive filter coefficient vector $[\hat{a}_1(n)\ \hat{b}_1(n)]^T$ and Figure 3 shows the trajectory followed by the composite factor during the adaptation process. Notice from those figures that the adaptive filter converges to the MSOE global optimal solution following characterized.

Figure 4 shows the mean composite square error (MCSE) performance surface for several values of the combining factor γ. Notice that as γ approaches one, the more quadratic and well behaved the MCSE function is, as opposed to the unbiased and multimodal surface associated to values of γ close to zero. Actually, for values in the interval $0 \leq \gamma \leq \approx 0.28$ the MSCE seems to be multimodal, and when $\gamma = 0$ the error function presents a local minimum at $[\hat{a}_1^*\ \hat{b}_0^*] = [-0.85\ -0.15]$ and a global minimum at $[\hat{a}_1^*\ \hat{b}_0^*] = [0.89\ 0.21]$.

6 Conclusions

In this work, we presented an alternative algorithm for adaptive IIR filtering based on composition of the well known output error and equation error basic schemes. The new algorithm, denominated the composite square error (CSE) algorithm, has an additional combination factor that allows control of the stability and bias characteristics of the resultant adaptation process. A new strategy for using a time-varying combination parameter was also introduced. Examples were included to demonstrate the positive results achieved with the proposed techniques.

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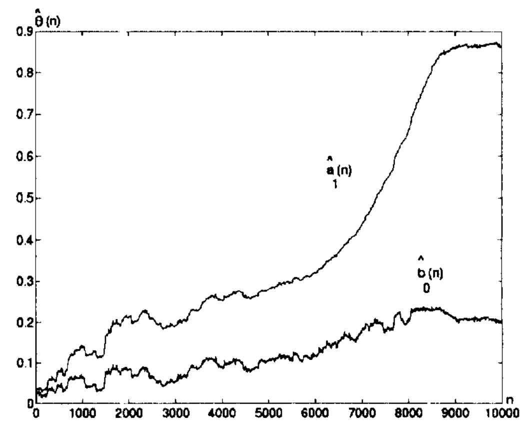

Figure 2: Adaptive Filter Coefficients Convergence

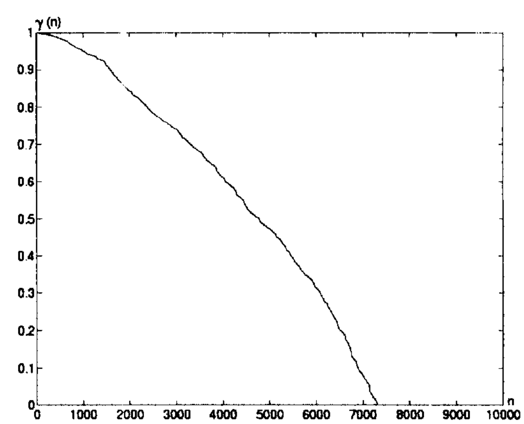

Figure 3: Composite Parameter Convergence

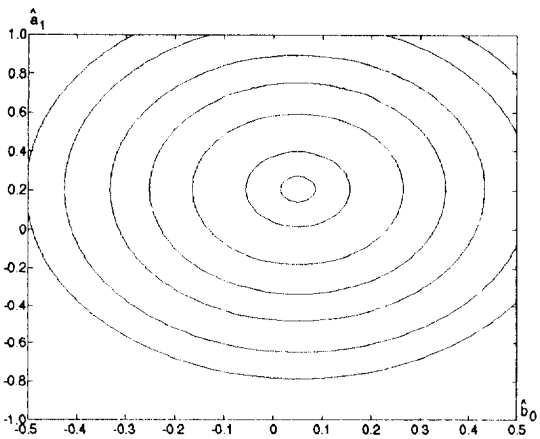

Figure 4: (A) MSCE Performance Surface - $\gamma = 1.0$

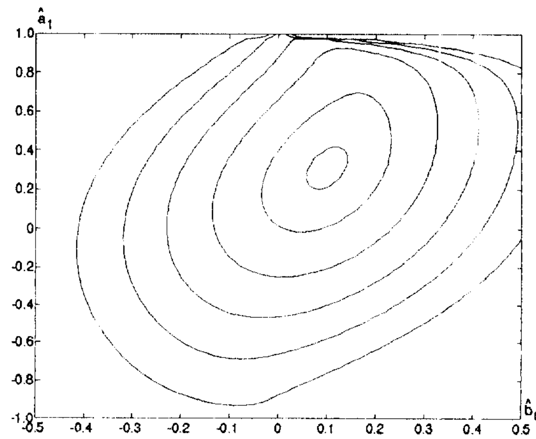

Figure 4: (B) MSCE Performance Surface - $\gamma = 0.6$

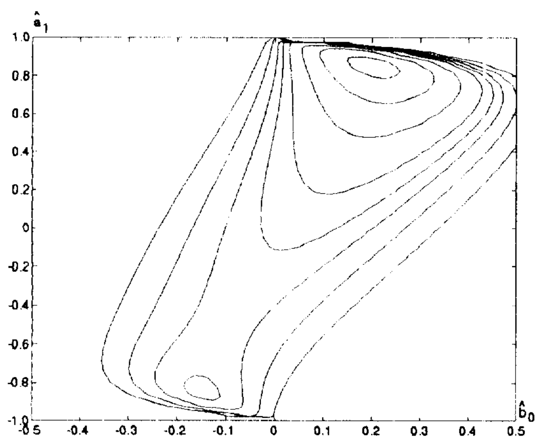

Figure 4: (C) MSCE Performance Surface - $\gamma = 0.2$

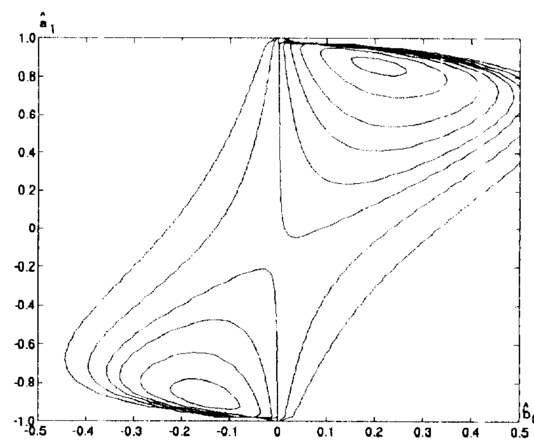

Figure 4: (D) MSCE Performance Surface - $\gamma = 0.0$