# Extension of the Equation Error Scheme for Alternative Realizations of Adaptive IIR Filters

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## ABSTRACT

The extension of the equation error (EE) adaptive algorithm to others than the direct-form realization is investigated. Implementing the EE algorithm with nondirect-form realizations is justified by the existence of EE-based adaptive algorithms that require continuous pole monitoring to avoid instability of the adaptive filter during the convergence process. Due to their respective importance, focus is given to the parallel, cascade, and lattice realizations. It is concluded that while the parallel and cascade structures present serious problems for a straightforward extension, the lattice realization is shown to be extremely suitable for efficient implementation of the EE algorithm.

## I. INTRODUCTION

The equation error (EE) method is a simple algorithm for updating the coefficients of an adaptive IIR filter [1-2] based on the EE signal defined as

$$e_E(n) = y(n)+\sum_{i=1}^{n_{\hat{a}}} \hat{a}_i(n)y(n-i)-\sum_{j=0}^{n_{\hat{b}}} \hat{b}_j(n)x(n-j)$$
$$= \hat{A}(q,n)[y(n)] - \hat{B}(q,n)[x(n)] \quad (1)$$

where $x(n)$ and $y(n)$ are respectively the input and desired output signals, and $\hat{B}(q,n)$ and $\hat{A}(q,n)$ are the difference polynomial operators respectively associated with the numerator and denominator of the direct-form transfer function of the adaptive filter. The standard EE algorithm is then described by the updating equation

$$\hat{\boldsymbol{\theta}}_d(n+1) = \hat{\boldsymbol{\theta}}_d(n)+\mu e_E(n)\hat{\boldsymbol{\phi}}_E(n) \quad (2)$$

where $\hat{\boldsymbol{\theta}}_d(n)$ is the direct-form coefficient vector defined as

$$\hat{\boldsymbol{\theta}}_d(n) = \left[\hat{a}_1(n)\ \ldots\ \hat{a}_{n_{\hat{a}}}(n)\ \hat{b}_0(n)\ \ldots\ \hat{b}_{n_{\hat{b}}}(n)\right]^T \quad (3)$$

$\mu$ is the convergence factor that controls the speed and stability of the overall adaptation scheme, and $\hat{\boldsymbol{\phi}}_E(n)$ is the EE information vector given by

$$\hat{\boldsymbol{\phi}}_E(n) = \left[y(n-1)\ldots y(n-n_{\hat{a}})\ x(n)\ldots x(n-n_{\hat{b}})\right]^T \quad (4)$$

It is important to notice that the composition of this information vector is obtained by differentiating the error signal with respect to each of the adaptive filter coefficients.

The basic properties of this EE adaptive algorithm are [1-2]:

- Quadratic and consequently unimodal error surface with respect to the coefficients of the adaptive filter;
- Guaranteed stable convergence for a known range of values of the convergence factor:

$$0 < \mu < \frac{2}{\hat{\boldsymbol{\phi}}_E^T(n)\hat{\boldsymbol{\phi}}_E(n)} \quad (5)$$

- Unbiased minimum in cases of absence of measurement and modelling perturbation noise and biased solution otherwise.

The first two properties represent positive aspects associated with the standard EE algorithm. The fact that this algorithm may converge to a suboptimal solution, however, constitutes a major drawback of this technique and has restricted its general use in many practical applications. As a consequence of that, several attempts have been made to modify and improve the EE algorithm resulting into new adaptive techniques such as the

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bias-remedy equation error [3] and the family of combined error algorithms of [4-5]. All these variants of the EE algorithm, however, do require some form of pole monitoring for the adaptive filter in order to assure stability of the overall adaptation process. As in practice the direct-form realization does not allow a computationally efficient pole monitoring procedure, the implementation of the EE algorithm with alternative realizations becomes a significant problem to be dealt with.

In this paper, we analyze the extension of the EE scheme for realizations distinct from the direct-form, concentrating our attention in the cascade, parallel, and lattice digital realizations.

## II. PARALLEL AND CASCADE EXTENSIONS

The output signals of a parallel and a cascade adaptive filters are respectively calculated as

$$\hat{y}_P(n) = \left\{ \sum_{i=1}^{N/2} \hat{H}_i(q,n) \right\} [x(n)] \tag{6a}$$

$$\hat{y}_C(n) = \left\{ \prod_{i=1}^{N/2} \hat{H}_i(q,n) \right\} [x(n)] \tag{6b}$$

where $N$ is the filter order, here assumed to be even, and $\hat{H}_i(q,n) = \frac{\hat{B}_i(q,n)}{\hat{A}_i(q,n)}$ is the transfer function of each individual block, which is considered to be of second order here, i.e.,

$$\hat{H}_i(q,n) = \frac{\hat{b}_{0i}(n) + \hat{b}_{1i}(n)q^{-1} + \hat{b}_{2i}(n)q^{-2}}{1 + \hat{a}_{1i}(n)q^{-1} + \hat{a}_{2i}(n)q^{-2}} \tag{7}$$

For the parallel and cascade structures, the corresponding numerator polynomials of the overall transfer functions are respectively given by

$$\hat{B}_P(q,n) = \sum_{i=1}^{N/2} \left\{ \hat{B}_i(q,n) \left[ \prod_{\substack{j=1 \\ j \neq i}}^{N/2} \hat{A}_j(q,n) \right] \right\} \tag{8a}$$

$$\hat{B}_C(q,n) = \prod_{i=1}^{N/2} \hat{B}_i(q,n) \tag{8b}$$

whereas for the overall denominator polynomials, we have that

$$\hat{A}_P(q,n) = \hat{A}_C(q,n) = \prod_{i=1}^{N/2} \hat{A}_i(q,n) \tag{9}$$

From equations (8) and (9), it is clear that in both cases of the parallel and cascade structures the EE signal defined in (1) is not a linear function of the adaptive filter coefficients. This results into nonquadratic and multimodal performance surfaces for these two particular cases. Such existence of multiple solutions when implementing an adaptive algorithm with the parallel and cascade realizations has already been reported in the literature by Nayeri and Jenkins in [6]. In fact, this is a consequence of the possibility of interchanging the order of the individual blocks without affecting the overall transfer function of the adaptive filter, keeping the value of the corresponding EE signal unaltered. In here, however, an even more complicated situation arises due to the nonlinearity of the EE signal with respect to the adaptive filter coefficients.

A possible solution for this problem guaranteeing unimodality of the resulting performance surface is to redefine the EE signal forcing a linear relationship with the filter coefficients. Such an example for the parallel and cascade realizations could be

$$e'_E(n) = \left\{ \sum_{i=1}^{N/2} \hat{A}_i(q,n) \right\} [y(n)] - \left\{ \sum_{i=1}^{N/2} \hat{B}_i(q,n) \right\} [x(n)] \tag{10}$$

A quick glance at this equation, however, reveals that the first derivative of this new error signal with respect to each coefficient $\hat{a}_{ji}$ or $\hat{b}_{ji}$ would be identical for all $i = 1, \ldots, N/2$, i.e.,

$$\frac{\partial e'_E(n)}{\partial \hat{a}_{jf}} = \frac{\partial e'_E(n)}{\partial \hat{a}_{jg}} \qquad j = 1,2;\ f,g = 1,\ldots,N/2 \tag{11a}$$

$$\frac{\partial e'_E(n)}{\partial \hat{b}_{jf}} = \frac{\partial e'_E(n)}{\partial \hat{b}_{jg}} \qquad j = 0,1,2;\ f,g = 1,\ldots,N/2 \tag{11b}$$

In this manner, the definition of the information vector would be the same for each of the composing blocks and an updating routine of the form

$$\hat{\boldsymbol{\theta}}(n+1) = \hat{\boldsymbol{\theta}}(n) + \mu e'_E(n) \hat{\boldsymbol{\phi}}'_E(n) \tag{12}$$

would drive all individual $N/2$ blocks to exactly the same point, making the adaptation process in this case totally meaningless.

Based on these points, it can be conjectured that the extension of the EE algorithm to either the parallel or the cascade of block structures does not constitute a practical alternative for the direct-form realization. In the next section, however, an efficient implementation of the EE algorithm using the lattice structure is introduced, allowing the stability of the adaptive filter to be monitored in real time.

## III. LATTICE EXTENSION

The output signal of a two-multiplier adaptive lattice filter is calculated as

$$F_i(n) = F_{i+1}(n) - \hat{k}_{i+1}(n)G_i(n-1); \; i = N-1, \ldots, 0 \quad (13a)$$

$$G_j(n) = G_{j-1}(n-1) + \hat{k}_j(n)F_{j-1}(n); \; j = 1, \ldots, N \quad (13b)$$

$$\hat{y}(n) = \sum_{j=0}^{N} \hat{h}_j(n)G_j(n) \quad (13c)$$

with $F_N(n) = x(n)$ and $G_0(n) = F_0(n)$, where the parameters $\hat{k}_i(n)$ and $\hat{h}_j(n)$ are the lattice coefficients and $N$ is the filter order.

In [7], it was verified that the $N$-order numerator and denominator polynomials of the direct-form transfer function $\hat{H}(q,n) = \frac{\hat{B}(q,n)}{\hat{A}(q,n)}$ can be calculated based on the lattice coefficients using the following recursive-in-order equations:

$$\hat{A}_m(q,n) = \hat{A}_{m-1}(q,n) + \frac{\hat{k}_m(n)}{\hat{k}_{m-1}(n)} \cdot \left[\hat{A}_{m-1}(q,n) - (1-\hat{k}^2_{m-1}(n))\hat{A}_{m-2}(q,n)\right]q^{-1} \quad (14a)$$

$$\hat{B}_m(q,n) = \hat{B}_{m-1}(q,n) + \frac{\hat{h}_m(n)}{\hat{k}_m(n)} \cdot \left[\hat{A}_m(q,n) - (1-\hat{k}^2_m(n))\hat{A}_{m-1}(q,n)\right] \quad (14b)$$

for $m = 2, \ldots, N$, with $\hat{A}_0(q,n) = 1$, $\hat{A}_1(q,n) = 1 + \hat{k}_1(n)q^{-1}$, and $\hat{B}_1(q,n) = (\hat{h}_0(n) + \hat{h}_1(n)\hat{k}_1(n)) + \hat{h}_1(n)q^{-1}$. With these equations, the EE signal can be calculated as defined in (1), using the lattice coefficients and therefore the implementation of the lattice-based EE algorithm can be performed as follows

$$\hat{\boldsymbol{\theta}}_\ell(n+1) = \hat{\boldsymbol{\theta}}_\ell(n) + \mu e_E(n)\hat{\boldsymbol{\phi}}_E(n) \quad (15)$$

where $\hat{\boldsymbol{\theta}}_\ell(n)$ is the lattice coefficient vector defined as

$$\hat{\boldsymbol{\theta}}_\ell(n) = \left[\hat{k}_1(n) \ldots \hat{k}_{n_{\hat{a}}}(n)\, \hat{h}_0(n) \ldots \hat{h}_{n_{\hat{b}}}(n)\right]^T \quad (16)$$

By using the same definition for the information vector as in equation (4), it is guaranteed that the above updating process is consistent with the standard direct-form algorithm in the sense that both methods yield a set of stationary points corresponding to the same input/output description for the adaptive filter.

## IV. SIMULATION

In this section, a computer simulation is presented using the lattice-based EE algorithm as described in the previous section in a system identification problem. Let the plant be defined as

$$H(q) = \frac{0.0154 + 0.0462q^{-1} + 0.0462q^{-2} + 0.0154q^{-3}}{1 - 1.99q^{-1} + 1.572q^{-2} - 0.4583q^{-3}} \quad (17)$$

what corresponds an optimal lattice coefficient vector equal to

$$\hat{\boldsymbol{\theta}}_\ell = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} = \begin{bmatrix} -0.87559 \\ 0.83546 \\ -0.45830 \\ 0.08565 \\ 0.14549 \\ 0.07685 \\ 0.01540 \end{bmatrix} \quad (18)$$

Consider an adaptive filter with $N = 3$ corresponding to a strictly sufficient order identification case and let the input signal be white noise with zero mean and unity variance. Assume also a perturbation signal statistically independent to the input signal consisting of white noise with zero mean and variance $\sigma_v^2 = 0.007$.

Fig. 1 and Fig. 2 show the convergence trajectories followed by the adaptive lattice coefficients updated by the EE algorithm with $\mu = 0.06$. It can be observed that due to the presence of a perturbation signal, the solutions achieved here were biased with respect to the optimal one represented by the dotted lines. A similar experiment without the presence of the perturbation signal was then executed for the lattice-based EE algorithms. As expected, in this case the lattice EE version converged to its global solution as depicted in Fig. 3 and Fig. 4, thus demonstrating the validity of the proposed method.

## V. CONCLUSION

The possibility of generalizing the equation error adaptive algorithm with nondirect-form structures was analyzed. The parallel and cascade of block structures were considered along with the two-multiplier lattice realization. It was concluded that the EE algorithm based on the parallel and cascade realizations does not constitute of a practical alternative for the direct form as the resulting performance surface was nonquadratic and multimodal. On the other hand, it was demonstrated the viability of implementing the EE algorithm based on the lattice structure allowing the stability of the adaptive filter to be determined on line. As a consequence, the practical implementation of several variations of the EE algorithm found in the literature becomes a concrete reality.

**REFERENCES**

[1] B. Widrow and S. Stearns, *Adaptive Signal Processing*, Prentice-Hall, Englewood Cliffs, New Jersey, USA, 1985.

[2] G. C. Goodwin and K. S. Sin, *Adaptive Filtering Prediction and Control*, Prentice-Hall, Englewood Cliffs, New Jersey, USA, 1984.

[3] J.-N. Lin and R. Unbenhauen, "Bias-remedy least mean square equation error algorithm for IIR parameter recursive estimation," *IEEE Trans. Signal Processing*, vol. 40, no. 1, pp. 62–69, Jan. 1992.

[4] S. L. Netto and P. S. R. Diniz, "Composite algorithms for adaptive IIR filtering," *IEE Electronic Letters*, vol. 28, no. 9, pp. 886–888, Apr. 1992.

[5] S. L. Netto and P. Agathoklis, "A new composite adaptive IIR algorithm," *Proc. 28th Asilomar Conf. Signals, Syst., and Computers*, Pacific Grove, USA, pp. 1506–1510, Oct./Nov. 1994.

[6] M. Nayeri and W. K. Jenkins, "Alternate realizations to adaptive IIR filters and properties of their performance surfaces," *IEEE Trans. Circuits Syst.*, vol. CAS-36, no. 4, pp. 485–496, Apr. 1989.

[7] S. L. Netto and P. Agathoklis, "Composite algorithms for adaptive IIR filtering using lattice realization," *Proc. IEEE Pacific Rim Conf. Communic., Comput., Signal Processing*, Victoria, Canada, pp. 189–192, May 1995.


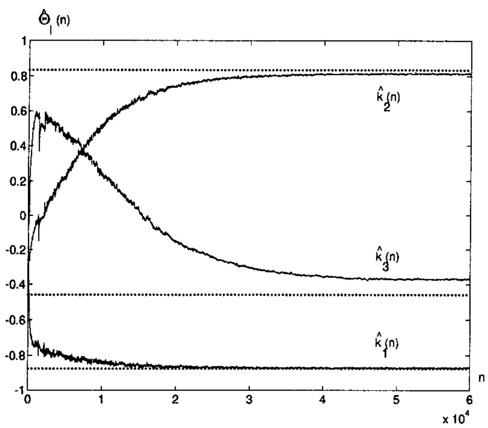


Fig. 1: Identification with noise - $\hat{k}(n)$ coefficients.

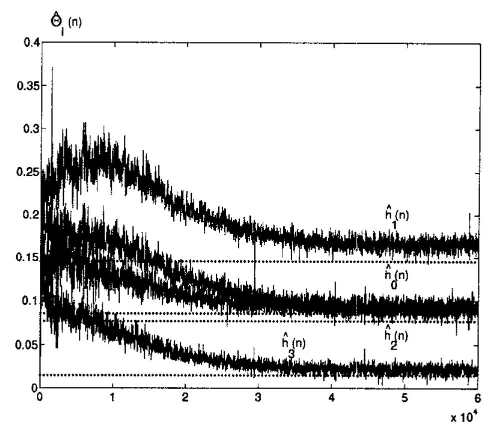


Fig. 2: Identification with noise - $\hat{k}(n)$ coefficients.

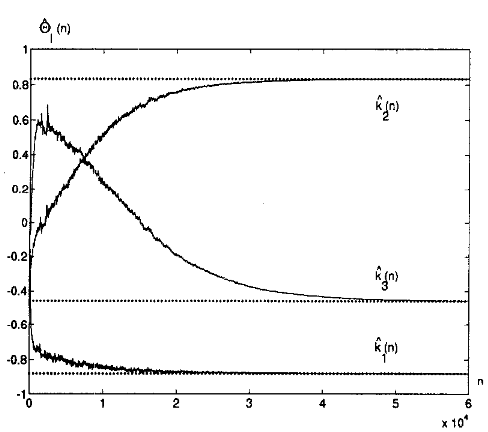


Fig. 3: Identification without noise - $\hat{k}(n)$ coefficients.

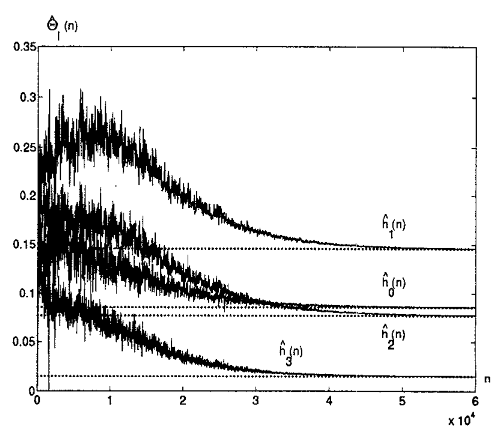


Fig. 4: Identification without noise - $\hat{h}(n)$ coefficients.