A Stable RLC Parameterization of the Direct Structure for Adaptive IIR Filters

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ABSTRACT

A stable parameterization of the direct-form structure with simple stability monitoring is introduced for adaptive IIR filtering. The proposed structure is obtained directly from passive *RLC* realizations. The stability of the resulting adaptive IIR filter is guaranteed by ensuring positive values for the equivalent passive *RLC* elements. Special attention is given to doubly terminated *RLC* networks to achieve optimal sensitivity properties of the overall transfer function with respect to parameter variations. Examples are included to demonstrate the application of the proposed technique.

I. INTRODUCTION

Adaptive IIR filters are a good alternative to adaptive FIR filters for modeling sharp resonances using significantly fewer coefficients [1]. The need for monitoring the poles of IIR filters to ensure stability, however, can be extremely costly, especially in cases where the adaptive filter is being implemented with the direct-form realization. This problem has motivated researchers to consider alternative forms for the adaptive filter realization. For instance, the parallel and cascade structures [1], [2] can be realized based on second-order transfer functions that can be easily checked for stability, but additional stationary points, introduced in the performance surfaces of these realizations by the over-parameterization, further reduce the convergence speed of the overall adaptation process. This work introduces a new parameterization for adaptive IIR filters that allows simple stability monitoring of the resulting realization during the algorithm execution, while keeping the number of parameters to a minimum. The proposed parameterization is obtained from passive *RLC*-circuit transfer functions through the bilinear transformation. In this way, bounded-input bounded-output stability is achieved as long as the adaptive filter parameters are positive. Additionally, optimal sensitivity with respect to parameter variation can be achieved if doubly terminated networks are used as initial passive circuits [3], [4]. This is a desirable characteristic in the context of adaptive IIR filters as it implies better stability properties in a limited-precision format, as will be later demonstrated via computer simulations.

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Another approach dealing with adaptive filter structures derived from passive networks is described by Forssén in [5]. This paper concentrates on a single specific digital filter realization and is, in consequence, less general in its content.

In the next section, the implementation of adaptive IIR filters based on the proposed realization is presented in a step-by-step easy to follow form. In Section III, a general discussion is included evaluating the convergence and implementation properties of the proposed structure. Simulations are included in Section IV in order to illustrate the validity and usefulness of the proposed technique.

II. PROPOSED PARAMETERIZATION

The starting point of the proposed adaptive IIR filter realization is a given passive *RLC* network of a specific order and spectrum characteristics (lowpass, highpass, etc.). Let $H(s)$ be the transfer function of this passive network, and $S_i(s)$ the sensitivity functions of $H(s)$ with respect to the value of each of the passive elements X_i, i.e.,

$$S_i(s) = \frac{\partial H(s)}{\partial X_i},\ i = 1, \ldots, P \tag{1}$$

where P is the number of passive elements of the network. By applying the bilinear transformation [6] to these functions, their discrete-time counterparts can be obtained as functions of X_i as

$$\hat{H}(z) = H(s)|_{s=\frac{2}{T}\frac{z-1}{z+1}} \tag{2a}$$

$$\hat{S}_i(z) = S_i(s)|_{s=\frac{2}{T}\frac{z-1}{z+1}} \tag{2b}$$

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for a given sampling period T. Based on these functions, the output-error (OE) adaptive algorithm [1] can then be implemented to update X_i and the stability of $\hat{H}(z)$ is guaranteed as long as the values of X_i are nonnegative.

A good option to use as basic passive circuit is the doubly terminated RLC network that is known to present optimal sensitivity of the filter with respect to variations in the values of its passive elements.

In order to illustrate the proposed technique, an easy-to-follow description is given here for the OE adaptive IIR algorithm based on either lowpass or highpass doubly terminated passive RLC networks. Other cases are easily derived as extensions to these steps.

Step 1: Given a passive network, obtain the corresponding transfer function. For the lowpass (LP) and highpass (HP) doubly-terminated networks (DTN) we have

$$H_{LP}(s) = \frac{1}{D_{LP}(s)} = \frac{1}{l_0 s^N + l_1 s^{N-1} + \cdots + l_N} \quad \text{(3a)}$$

$$H_{HP}(s) = \frac{s^N}{D_{HP}(s)} = \frac{s^N}{h_0 s^N + h_1 s^{N-1} + \cdots + h_N} \quad \text{(3b)}$$

where N is the order of the given transfer function, and the coefficients l_j and h_j are functions of the passive elements X_i of the passive network.

Step 2: Generate the corresponding discrete-time transfer function $\hat{H}(z)$ by applying the bilinear transformation to $H(s)$. This step is easily accomplished by noticing that the LP-DTN and the HP-DTN will present discrete-time transfer functions of the form

$$\hat{H}_{LP}(z) = \frac{(z+1)^N}{\hat{D}(z)} = \frac{(z+1)^N}{\hat{l}_0 z^N + \hat{l}_1 z^{N-1} + \cdots + \hat{l}_N} \quad \text{(4a)}$$

$$\hat{H}_{HP}(z) = \frac{(z-1)^N}{\hat{D}(z)} = \frac{(z-1)^N}{\hat{h}_0 z^N + \hat{h}_1 z^{N-1} + \cdots + \hat{h}_N} \quad \text{(4b)}$$

respectively, where the coefficients $\hat{l}_j$ and $\hat{h}_j$ are obtained from their continuous-time counterparts as

$$\begin{bmatrix} \hat{l}_0 \\ \hat{l}_1 \\ \vdots \\ \hat{l}_N \end{bmatrix} = \mathbf{M} \begin{bmatrix} l_0 \\ l_1 \\ \vdots \\ l_N \end{bmatrix} ; \quad \begin{bmatrix} \hat{h}_0 \\ \hat{h}_1 \\ \vdots \\ \hat{h}_N \end{bmatrix} = \mathbf{M} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_N \end{bmatrix} \quad (5)$$

The $(N+1)\times(N+1)$ matrix $\mathbf{M}$ implements the corresponding bilinear transformation to a N^{th}-order polynomial.

Step 3: The derivation of the discrete-time sensitivity functions is greatly simplified by noticing that for both the LP and HP networks these functions can be written as

$$\hat{S}_i(z) = \frac{\hat{N}_i(z)}{\hat{D}(z)} \hat{H}(z)$$

$$= \frac{\hat{n}_{i0} z^N + \hat{n}_{i1} z^{N-1} + \cdots + \hat{n}_{iN}}{\hat{D}(z)} \hat{H}(z) \quad (6)$$

where the coefficients of $\hat{N}_i(z)$ are calculated as

$$\begin{bmatrix} \hat{n}_{i0} \\ \hat{n}_{i1} \\ \vdots \\ \hat{n}_{iN} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \frac{\partial l_0}{\partial X_i} \\ \frac{\partial l_0}{\partial X_i} \\ \vdots \\ \frac{\partial l_0}{\partial X_i} \end{bmatrix} ; \quad \begin{bmatrix} \hat{n}_{i0} \\ \hat{n}_{i1} \\ \vdots \\ \hat{n}_{iN} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \frac{\partial h_0}{\partial X_i} \\ \frac{\partial h_0}{\partial X_i} \\ \vdots \\ \frac{\partial h_0}{\partial X_i} \end{bmatrix} \quad (7)$$

These relationships are readily obtained by applying the sensitivity function definition (2b) directly to equations (4a) and (4b).

Step 4: Based on the discrete-time transfer function and sensitivity functions given in equations (4) and (6), the OE adaptive IIR algorithm [1] can be used to update the adaptive-filter parameter vector composed of the passive elements X_i, i.e., $\hat{\boldsymbol{\theta}}(n) = [X_1(n) \ \cdots \ X_P(n)]^T$. This algorithm is based on the output error signal given by

$$e_{OE}(n) = y(n) - \hat{H}(q)\{x(n)\} \quad (8)$$

where q is the linear unit-delay operator defined as

$$q^k\{x(n)\} = x(n-k), \ k \in \mathbb{Z} \quad (9)$$

and $x(n)$ and $y(n)$ are, respectively, the input signal and the desired output signal for the adaptive filter. Also, the gradient vector associated with the OE algorithm is given by

$$\hat{\boldsymbol{\nabla}}_{OE}(n) = \frac{\partial e_{OE}^2(n)}{\partial \hat{\boldsymbol{\theta}}} = -e_{OE}(n)\hat{\boldsymbol{\phi}}_{OE}(n) \quad (10)$$

where $\hat{\boldsymbol{\phi}}_{OE}(n)$ is the information vector formed by the sensitivity functions with respect to each of the elements of $\hat{\boldsymbol{\theta}}(n)$, i.e.,

$$\hat{\boldsymbol{\phi}}_{OE}(n) = \left[\hat{S}_1(q)\{x(n)\} \ \cdots \ \hat{S}_P(q)\{x(n)\}\right]^T \quad (11)$$

From equation (6), it can be verified that the information vector for the LP and HP doubly terminated networks can be simplified to the form

$$\hat{\boldsymbol{\phi}}_{OE}(n) = \left[\frac{\hat{N}_1(q)}{\hat{D}(q)}\{\hat{y}(n)\} \ \cdots \ \frac{\hat{N}_P(q)}{\hat{D}(q)}\{\hat{y}(n)\}\right]^T \quad (12)$$

which is more efficient to be implemented in real time.

Using (8) and (7), the OE algorithm based on a steepest-descent approach [7] is then implemented for the proposed stable parameterization as

$$\hat{\boldsymbol{\theta}}(n+1) = \hat{\boldsymbol{\theta}}(n) + \mu e_{OE}(n)\hat{\boldsymbol{\phi}}_{OE}(n) \quad (13)$$

where μ is a convergence factor that controls the speed and stability of the overall adaptation process. The stability of the resulting structure can then be guaranteed by assuring that the values of the adaptive filter parameters X_i remain nonnegative.

III. DISCUSSION

The most immediate advantage related to the passive parameterization is the fact that the filter stability is easily guaranteed as long as the elements of $\hat{\boldsymbol{\theta}}(n)$, X_i, remain nonnegative. Additionally, as the stability monitoring is carried out by checking each coefficient individually, update stalemate is less likely to occur than with methods that require $\hat{\boldsymbol{\theta}}(n+1) = \hat{\boldsymbol{\theta}}(n)$ once instability is detected.

As mentioned before, the approach may lead to a very low-sensitivity parameterization. This fact is an interesting property for adaptive IIR filters when modeling poles close to the unity circle, for variations in the parameters have a reduced effect on the pole location. The low sensitivity together with the simple stability-checking routine impart robustness to the algorithm in a real-time implementation.

It should be noted that the passive network used as a design starting point will carry its properties to the adaptive filter. Different networks like lowpass, highpass, bandpass, or bandstop circuits can be used according to the application requirements. In channel equalization [8], for example, the adaptive filter usually has highpass characteristics. In general, the designer can rely on prior knowledge of the problem at hand in order to properly select the initial passive network.

IV. SIMULATIONS

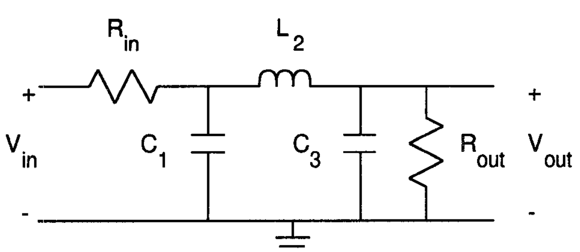

Figure 1: Third-Order RLC Passive Circuit

Before the computer simulations are described, let us illustrate how the method can be applied to a passive lowpass structure. Consider the passive RLC circuit depicted in Fig. 1. Assuming that $R_{in} = R_{out} = 1\Omega$, the circuit is described by the transfer function $H(s) = V_{out}/V_{in}$ given by

$$H(s) = \frac{1}{D(s)} = \frac{1}{a_0 s^3 + a_1 s^2 + a_2 s + a_3} \tag{14}$$

with $a_0 = (C_1 L_2 C_3)$, $a_1 = (C_1 L_2 + L_2 C_3)$, $a_2 = (C_1 + L_2 + C_3)$, and $a_3 = 2$.

The sensitivity functions of $H(s)$ with respect to the inductors and capacitors are

$$S_1(s) = \frac{\partial H(s)}{\partial C_1} = \frac{-L_2 C_3 s^3 - L_2 s^2 - s}{D^2(s)} \tag{15a}$$

$$S_2(s) = \frac{\partial H(s)}{\partial L_2} = \frac{-C_1 C_3 s^3 - (C_1 + C_3)s^2 - s}{D^2(s)} \tag{15b}$$

$$S_3(s) = \frac{\partial H(s)}{\partial C_3} = \frac{-C_1 L_2 s^3 - L_2 s^2 - s}{D^2(s)} \tag{15c}$$

with $D(s)$ defined as in (14).

The discrete-time transfer function is obtained through the bilinear transformation which, for $T = 2$, gives

$$\hat{H}(z) = \frac{\hat{N}(z)}{\hat{D}(z)} = \frac{z^3 + 3z^2 + 3z + 1}{\hat{a}_0 z^3 + \hat{a}_1 z^2 + \hat{a}_2 z + \hat{a}_3} \tag{16}$$

with

$$\begin{bmatrix} \hat{a}_0 \\ \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix} = \mathbf{M} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \mathbf{M} \begin{bmatrix} C_1 L_2 C_3 \\ (C_1 + C_3) L_2 \\ C_1 + L_2 + C_3 \\ 2 \end{bmatrix} \tag{17}$$

where $\mathbf{M}$ is the bilinear transformation matrix given in this third-order case by

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -3 & -1 & 1 & 3 \\ 3 & -1 & -1 & 3 \\ -1 & 1 & -1 & 1 \end{bmatrix} \tag{18}$$

Similar operations performed on the sensitivity functions result in

$$\hat{S}_1(z) = \frac{\hat{N}_1(z)}{\hat{D}(z)} \frac{\hat{N}(z)}{\hat{D}(z)} = \frac{\hat{a}_{10} z^3 + \hat{a}_{11} z^2 + \hat{a}_{12} z + \hat{a}_{13}}{\hat{D}(z)} \hat{H}(z) \tag{19a}$$

$$\hat{S}_2(z) = \frac{\hat{N}_2(z)}{\hat{D}(z)} \frac{\hat{N}(z)}{\hat{D}(z)} = \frac{\hat{a}_{20} z^3 + \hat{a}_{21} z^2 + \hat{a}_{22} z + \hat{a}_{23}}{\hat{D}(z)} \hat{H}(z) \tag{19b}$$

$$\hat{S}_3(z) = \frac{\hat{N}_3(z)}{\hat{D}(z)} \frac{\hat{N}(z)}{\hat{D}(z)} = \frac{\hat{a}_{30} z^3 + \hat{a}_{31} z^2 + \hat{a}_{32} z + \hat{a}_{33}}{\hat{D}(z)} \hat{H}(z) \tag{19c}$$

with the $\hat{a}_{ij}$ coefficients given by

$$\begin{bmatrix} \hat{a}_{10} \\ \hat{a}_{11} \\ \hat{a}_{12} \\ \hat{a}_{13} \end{bmatrix} = M \begin{bmatrix} -L_2 C_3 \\ -L_2 \\ -1 \\ 0 \end{bmatrix} \tag{20a}$$

$$\begin{bmatrix} \hat{a}_{20} \\ \hat{a}_{21} \\ \hat{a}_{22} \\ \hat{a}_{23} \end{bmatrix} = M \begin{bmatrix} -C_1C_3 \\ -(C_1+C_3) \\ -1 \\ 0 \end{bmatrix} \quad (20b)$$

$$\begin{bmatrix} \hat{a}_{30} \\ \hat{a}_{31} \\ \hat{a}_{32} \\ \hat{a}_{33} \end{bmatrix} = M \begin{bmatrix} -C_1L_2 \\ -L_2 \\ -1 \\ 0 \end{bmatrix} \quad (20c)$$

respectively.

The OE adaptive IIR algorithm can be used to update the parameter-vector

$$\hat{\boldsymbol{\theta}}(n) = [C_1(n)\ L_2(n)\ C_3(n)]^T \quad (21)$$

as described in the previous section.

Example: In order to investigate the convergence characteristics of the proposed algorithm, a system identification scenario was created where the plant is described by

$$H(z) = \frac{z^3 + 3z^2 + 3z + 1}{2.321z^3 + 6.277z^2 + 5.683z + 1.719} \quad (22)$$

The proposed stable parameterization was compared to the direct-form structure implementation [1] in a 60000-point simulation using 32-bit floating-point arithmetic. In both cases the OE algorithm was used to adapt the initially relaxed denominator coefficients, whereas the numerator coefficients were set to their optimal values. The input signal was a zero-mean white Gaussian noise with variance equal to 0.05. Fig. 2 shows the results of the experiment. It can be clearly seen in this figure that the conventional direct-form structure implementation diverged even with a very small value of μ, in this case equal to 0.0004. Other tested values of μ resulted in eventual instability or extremely slow convergence rates. On the other hand, for the proposed parameterization μ was chosen equal to 0.16 to achieve maximum speed of convergence without causing instability. Stability checks were not implemented.

V. CONCLUSION

A new stable parameterization for IIR adaptive filters was introduced. The proposed approach relates the discrete-time filter with a continuous-time passive realization counterpart, which results in a simple stability-monitoring routine. Furthermore, for certain passive structures the optimal sensitivity of the transfer function with respect to parameter variations impart extra robustness to the method. The closer to the unit circle the poles of the filter are, the more advantageous the low sensitivity is. An example was shown where the proposed method is seen to compare very favorably to the direct-form structure adaptive filter, for the latter diverged.

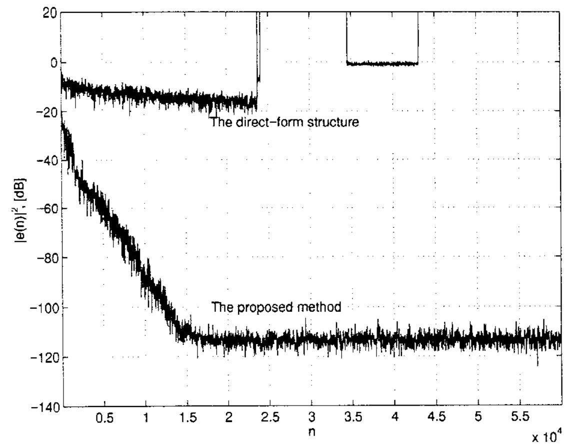

Figure 2: Convergence in time of the square value of output error signal.

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