# ORTHONORMAL ADAPTIVE IIR FILTER WITH POLYPHASE REALIZATION

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## ABSTRACT

In this work, a particular realization for adaptive IIR filtering and a related special interpretation of the Steiglitz-McBride method are presented. The proposed realization is based on an orthonormal family of functions. The intrinsic unstable behavior of adaptive IIR filters when modeling highly underdamped poles is avoided by visualizing the orthogonal filter as a particular polyphase realization. A stationary point analysis of the resulting adaptive filter shows that it maintains a one-to-one coefficient mapping with the direct-form realization. In this manner, many known properties of the SM algorithm related to the direct-form realization can be extended to the proposed realization. Computer simulations are given illustrating the good performance of the proposed adaptive IIR filter with respect to computational complexity, modeling capability, and reduced order performance. The main application emphasis is oriented to echo cancellation, but other areas of adaptive signal processing can also be considered.

## 1. INTRODUCTION

There is a wide variety of applications with a clear requirement for low complexity adaptive filters. The fact that IIR filters are useful in adaptive signal processing has been widely explored in the last years. It is expected that these adaptive filters improve the performance of their FIR counterpart in many areas, as, for example, echo cancellation or postcursor equalization in x-DSL receivers.

In fact, often echo cancellation in many high speed communication systems have channel model where many underdumped poles appear. A similar discussion applies in decision feedback equalization where the postcursor filter, normally a long FIR filter, is modeling the postcursor (causal) intersymbol interference of the channel [7]. An important aspect of this application is that the dynamic of the channel to be equalized is mainly dominated by two or three real poles [9]. In this case the substitution of a single FIR adaptive filter for one with a IIR realization is expected to introduce considerable savings in filter complexity.

The main problems related to adaptive IIR filters seem to be avoiding the suboptimum behavior of a stochastic gradient algorithm and eliminating possible unstability of the adaptive filter when underdumped poles are being modeled. A solution for the first problem is obtained by changing the criterium to be minimized from the mean-squared output error (MSOE) to the Steiglitz-McBride (SM) approach. Although global convergence is still not easy to be fully established, an important result relating global MSOE minima and the SM minima [5], even for insufficient order cases, is the main evidence that this method is useful in general cases.

Perhaps the main drawback related to the SM method is that stability of the estimates can not be guaranteed. In fact, this problem is far more difficult because instability of adaptive IIR filters when modeling underdamped poles seems to be an structural problem [2]. However recently an interesting analysis of the so-called *polyphase realization* [3] has shown that a simple mapping on denominator polynomial can strongly attenuate such unstable behavior.

By addressing the idea of suitable modeling related to IIR filters, the main focus of this work is oriented to the introduction of a realization based on a family of orthonormal rational functions, that also serves for an efficient implementation of the SM algorithm. Such realization can be reduced, as a special case, to a realization based on the Laguerre functions or the Kautz functions [4]. Amongst the advantages of the new realization, we can mention that several distinct or multiple order poles can be used, the stability test is simple, the extension of the polyphase concept to the proposed realization is easy, and also that nice properties in terms of modularity and suitable numerical conditioning can be verified. Overall, it is important to highlight the simplicity of the proposed adaptive filter, as the resulting computational complexity is $O(N)$, where $N$ is the order of the adaptive realization. Also, trigonometric computations are not required, as opposed to the orthonormal lattice filter in [6].

The paper is organized as follows: in section II a review of both the proposed orthonormal realization for the adaptive IIR filter and their utilization with the SM algorithm is included. In section III stationary points and convergence are discussed. In section IV a polyphase extension of the proposed scheme is pre-

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sented. In section V examples of the utilization of the proposed adaptive filter are included.

## 2. EFFICIENT ADAPTIVE IIR FILTERING

To introduce the orthonormal realization, suppose that a proper stable linear time invariant dynamical system is described by $y(n) = G(q)u(n)$, where $u(n)$ is the input signal and $y(n)$ is the output signal. The transfer function $G(q) = \frac{B(q)}{A(q)}$, with $B(q) = b_0 + b_1 q^{-1} + \ldots + b_M q^{-M}$ and $A(q) = 1 + a_1 q^{-1} + \ldots + a_N q^{-N}$, is assumed to be strictly causal, i.e. $M < N$, and asymptotically stable, and so the model belongs to $\mathcal{H}_2$. In this manner, it is possible to write that $G(q) = \sum_{i=0}^{N} \nu_i F_i(q)$, where $\{F_i(q)\}_0^N$ is an orthonormal family in $\mathcal{H}_2$. If the system modeled is rational, an FIR orthonormal family will have a poor performance in the identification of the parameters of the true system. In such case, it is desirable that $F(q)$ is a family of rational functions, as for example the Laguerre basis whose orthonormal functions are identical first-order sections. However, a crucial limitation of the Laguerre basis is that no more than one mode per base function can be incorporated in the resulting function representation. A natural extension is to consider basis functions builded from distinct (real or conjugate complex) poles, as for example in the structure presented in [1] [4].

In this paper, we focus our interest in the case of real poles for two main reasons. The first one is related to the applications considered. In fact, both echo cancellation and postcursor-decision-feedback equalization can afford such simplified model without any major loss of performance. The second reason is related to the properties of the updating algorithm as discussed below.

Then, for the particular case of interest, the basis functions can be written as

$$F_i(q) = \frac{\alpha_i}{1 - \beta_i q^{-1}} \prod_{k=1}^{i-1} \frac{q^{-1} - \beta_k}{1 - \beta_k q^{-1}} \tag{1}$$

where $\beta_i$ is the $i$-th pole and $\alpha_i = \sqrt{1 - \beta_i^2}$ is a normalization constant. Fig. 1 depicts the proposed realization, highlighting the fact that when multiple-order poles need to be modeled, all-pass sections should be included in the last branch as necessary. Note that high order sections can also be considered to model complex conjugate poles, but complexity (modularity) is generally higher.

This given construction preserves orthonormality and also provides a unifying formulation of all known system-identification orthonomal structures like FIR, Laguerre, or Kautz [4] bases. In some applications, like in postcursor equalization for x-DSL systems [9], the order of the system is well known but the value of their poles and their zeros not.

Since the proposed realization is orthonormal, the structural interpretation of the SM algorithm as the

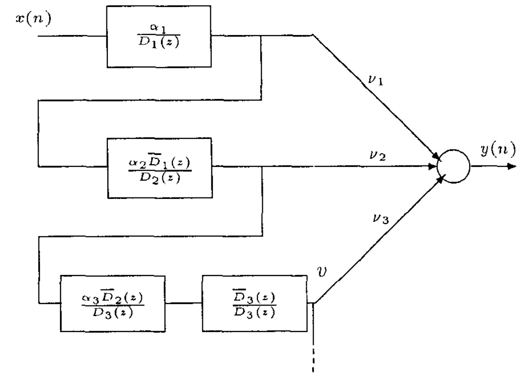


Figure 1: Proposed adaptive IIR filter with orthonormal realization.

presented in [6] is useful to obtain an efficient updating algorithm. The main results related to this development are studied in [8]. The main aspects are the following:

Given the orthonormal function set represented by matrix $\boldsymbol{Q}$ as

$$\begin{pmatrix} \boldsymbol{x}(n) \\ r(n) \end{pmatrix} = \boldsymbol{Q} \begin{pmatrix} \boldsymbol{x}(n-1) \\ u(n) \end{pmatrix}; \qquad \boldsymbol{Q} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{b} \\ \boldsymbol{g} & \rho \end{pmatrix}$$

where

$$\boldsymbol{x}(n) = [F_1(q)u(n) \; F_2(q)u(n) \; \ldots \; F_N(q)u(n)]^T \tag{2}$$

and the apriori error $e_p(n)$ defined as

$$e_p(n) = d(n) - y(n) \tag{3}$$

$$d(n) = [0 \; 0 \; \ldots \; 1]\boldsymbol{Q}^{-1} \begin{pmatrix} \boldsymbol{t}(n) \\ s(n) \end{pmatrix} \tag{4}$$

where $t_i(n) = F_i(q)d(n)$, and $\boldsymbol{t}(n)$ and $s(n)$ are given by

$$\begin{pmatrix} \boldsymbol{t}(n) \\ s(n) \end{pmatrix} = \boldsymbol{Q} \begin{pmatrix} \boldsymbol{t}(n-1) \\ d(n) \end{pmatrix}$$

Then the algorithm can be summarized as

$$\boldsymbol{\theta}(n) = \boldsymbol{\theta}(n-1) + \mu \boldsymbol{\psi}_o(n) e_p(n-1) \tag{5}$$

$\boldsymbol{\psi}_o(n) = [F_1(q)u(n) \ldots F_N(q)u(n) \; \lambda_1 F_1(q)d(n-1) \ldots \lambda_N F_N(q)d(n-1)]^T$, $\mu$ is a positive constant, and $\lambda_i = (-1)^i \frac{\alpha_i(n-1)}{\prod_{k=1}^{i-1} \beta_k}$.

The second reason that justify an adaptive filter realization modeling only real poles can now be explained. The SM algorithm can approximate very closely the global MSOE even in reduced order settings [5]. In the worst case of reduced order identification (i.e., complex conjugate pole pairs), that is indeed the expected behavior. Hence, the combination of its computational simplicity and suitable modeling for the addressed applications makes the proposed constrained realization an excellent candidate to solve the problems commonly associated to adaptive IIR filters.

## 3. STATIONARY POINT ANALYSIS

The existence of manifolds [5] is an important issue in an adaptive IIR realization for unambiguous stationary point definition. By straightforward algebraic manipulations it is easy to show that, in order to implement the equivalent direct-form transfer function $\frac{\hat{B}(q)}{\hat{A}(q)}$, the regressor vector can be written as

$$\psi_d(n)=[\tfrac{1}{\hat{A}(q)}u(n)\dots\tfrac{1}{\hat{A}(q)}u(n-N)\ \tfrac{q^{-1}}{\hat{A}(q)}d(n)\dots\tfrac{q^{-N}}{\hat{A}(q)}d(n)]^T \quad (6)$$

The stationary points for both the direct-form realization and the orthonormal realization are the solution of the following equation

$$E\{\psi_d(n)e_p(n)\} = E\{\psi_o(n)e_p(n)\} = 0 \quad (7)$$

Clearly, if a non singular matrix mapping $T$ exists for the direct-form and orthonormal realizations, such that $\psi_o(n) = T\psi_d(n)$, all direct-form stationary points are maintained in the new realization. If the mapping is singular, however, additional stationary points and corresponding manifolds are expected. This is true, for instance, for the parallel and cascade realizations where, for an equivalent model based on first-order sections, the manifolds exist when the poles of two different adaptive sections are equal [5].

It is not hard to see that for the proposed constrained realization, the matrix $T$ can be written as

$$T = \begin{bmatrix} T_1 & 0 \\ 0 & T_1 \end{bmatrix}$$

where

$$T_1 = \begin{bmatrix} \alpha_1 & -\alpha_1\sum_{k=2}^{N} a_k & \alpha_1\sum_{k=2}^{N}\prod_{j=N}^{2} a_k a_{k-j} & \cdots & \alpha_1\prod_{k=2}^{N} a_k \\ & & \vdots & & \\ \alpha_N\prod_{k=1}^{N-1} a_k & \cdots & \alpha_N\sum_{k=1}^{N-1}\prod_{j=N-1}^{1} a_k a_{k-j} & -\alpha_N\sum_{k=1}^{N-1} a_k & \alpha_N \end{bmatrix}$$

the determinant of which, by induction, is given by

$$\det(T_1) = \left(\prod_{k=1}^{N}\alpha_k\right)\left(\prod_{k,j,k\neq j}^{N} 1 - a_k a_j\right)$$

Hence, since $|a_k| < 1$, for $k = 1,\dots,N$, this determinant is never null. Then the stationary points of the orthonormal realization present a one-to-one correspondence to the stationary points of the direct-form realization.

If we consider the special case of high-order poles in the proposed realization, when they are contemplated in the adaptive filter structure, as in Fig. 1, a similar conclusion can be obtained. This is easy to see by including the all-pass first-order sections to each output tap of the orthonormal realization as needed. Assuming, for instance, a third-order realization, with a single pole $a_1$, and a second-order pole $a_2$, with $a_1 \neq a_2$, then the matrix $T_1$ takes the form

$$T_1 = \begin{bmatrix} \alpha_1 & -2a_2\alpha_1 & a_2^2\alpha_1 \\ a_1a_2\alpha_2 & -(a_1+a_2)\alpha_2 & \alpha_2 \end{bmatrix}$$

Since the direct-form realization uses in general 3 coefficients, a direct map can not be obtained. However, as can be easily verified the rows of $T_1$ are linearly independent defining a suitable definition of the associated gradients.

As a consequence of the previous analysis, the powerful results related to MSOE scheme also apply to the SM updating algorithm using the proposed realization [5].

## 4. POLYPHASE ORTHONORMAL REALIZATION

In this section, it is shown that the orthonormal realization is proper to the implementation of the polyphase adaptive IIR filter. The main idea here is to improve the numerical conditioning of the coefficient covariance matrix, when underdumped poles are present, by overmodelling. This is achieved mapping the denominator dynamic of the filter to a radius lower than one, i.e., by replacing $z$ by $z^p$, with $p$ a integer [3]. In this manner, the polyphase realization can be written as

$$H(z) = \frac{B_p(z)}{A_p(z^p)} = \frac{b_{p,0} + b_{p,1}z^{-1} + \dots + b_{p,pN}z^{pN}}{1 - a_{p,1}z^{-p} - \dots - a_{p,N}z^{pN}}$$

where $p$ is the polyphase expansion factor.

For the case of the orthonormal realization it can be shown that a special form of the poles leads naturally to the polyphase realization. This can be shown for $p$ an even integer, particularly it is shown for second order section. An straightforward proof can be obtained for higher even $p$ expansion factors. After some elaboration, the orthonormal realization with second order sections can be written as

$$F_k(z) = \frac{\alpha_{k0} + \alpha_{k1}z^{-1}}{1 - a_{k1}z^{-1} - a_{k2}z^{-2}} \quad (8)$$

$$F'_k(z) = \frac{\alpha'_{k0} + \alpha'_{k1}z^{-1}}{1 - a_{k1}z^{-1} - a_{k2}z^{-2}} \quad (9)$$

where in order to maintain orthonormality, i.e., $||F_k(z)|| =$ 1 and $< F_k(z), F_l(z) >= \delta_{k,l}$, the numerator coefficients are given by

$$\alpha_{k0} = -\frac{1}{2}\sqrt{c_1}(\sqrt{c_2} + \sqrt{c_3}); \qquad \alpha_{k1} = \frac{1}{2}\sqrt{c_1}(\sqrt{c_2} - \sqrt{c_3}$$

$$\alpha'_{k0} = \alpha_{k1}; \qquad \alpha'_{k1} = \alpha_{k0}$$

with $c_1 = 1 - a_{k2}$, $c_2 = 1 - a_{k1} + a_{k2}$, and $c_3 = 1 + a_{k1} + a_{k2}$. Then, because $a_{k1} = 0$ for a polyphase factor of 2, it is easy to show that $\alpha_{k0} = \sqrt{1 - a_{k2}^2}$, $\alpha_{k1} = 0$, $\alpha'_{k0} = 0$, and $\alpha'_{k1} = \sqrt{1 - a_{k2}^2}$. Finally equations (8) and (9) can be rewritten as

$$F_k(z) = \frac{\alpha_{k0}}{1 - a_{k2}z^{-2}}; \qquad F'_k(z) = F_k(z)z^{-1}$$

respectively. In the same way, for fourth order sections ($p = 4$), with $a_{k1} = a_{k2} = a_{k3} = 0$, it is possible to obtain

$$F_k(z) = \frac{\alpha_{k0}}{1 - a_{k4}z^{-4}}; \qquad F'_k(z) = F_k(z)z^{-1}$$
$$F''_k(z) = F_k(z)z^{-2}; \qquad F'''_k(z) = F_k(z)z^{-3}$$

This procedure can continue in a similar form for higher and higher order expansion factors. The important remark to make here is related to the condition on the poles of each section: If they are different, then all the stationary point of the direct-form (real-pole) polyphase realization are maintained. Also it is important to note that all properties related to the orthonormal realization are maintained, i.e., a suitable scaling of all the variables and input orthogonalization.

## 5. COMPUTER SIMULATIONS

To illustrate the expected behavior of the proposed realization, a comparison between the orthonormal SM realization and the polyphase orthonormal SM realization in a system identification application is presented. The plant to be identified is $H(z)=\frac{0.3-0.4z^{-1}}{1-1.75z^{-1}-0.76z^{-2}}$, the input is unit variance white noise. A white noise was included to introduce a SNR equal to 50 dB. The results of the simulations are depicted in figure 2, where an average over 100 computer runs was performed. As illustrated in this figure, even for this example of high-module poles (0.9 and 0.85), the performance of both adaptive filter realizations presents fast convergence with better behavior, as expected, obtained with the polyphase version of 4-th order.

## 6. CONCLUSIONS

A new scheme for adaptive IIR filtering is proposed based on an orthonormal family of functions yielding a one-to-one correspondence to the direct-form structure. A Steiglitz-McBride algorithm is used to achieve (quasi-)optimal performance, even in cases of insufficient order modeling. It was verified that the adaptive IIR orthonormal realization used with the restricted (real-pole) Steiglitz-Mcbride algorithm presents interesting characteristics as low computational complexity, good convergence speed and excellent numerical conditioning.

With the complementary results obtained by computer simulations, it was possible to conclude that this structure performs as required by many high speed applications, as for example echo cancellation and equalization in broadband transmission systems.

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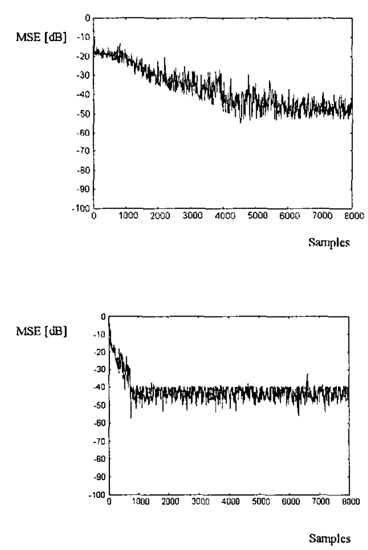


Figure 2: Learning curve for orthonormal and polyphase orthonormal adaptive IIR filter with SM algorithm.