

DESIGN OF FIR FILTERS COMBINING THE FREQUENCY-RESPONSE MASKING AND THE WLS-Chebyshev APPROACHES

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ABSTRACT

This paper presents an efficient method for designing FIR filters by combining the frequency-response masking (FRM) approach with the WLS-Chebyshev method. With the proposed method, the specifications for the base filter in the FRM approach can be relaxed. This is accomplished based on the ability of the WLS-Chebyshev method for finding a good compromise between the minimum attenuation and total energy within the filter's stopband. A typical design performed with the proposed method is included indicating that the total number of multiplications per output sample required is significantly reduced.

1. INTRODUCTION

The frequency-response masking (FRM) approach is a very efficient alternative for designing linear-phase FIR digital filters with large passbands and sharp transition bands. With such method, allowing an increase of the filter delay time, it is possible to reduce the filter complexity (number of multipliers and adders required per output sample) when compared to the standard design methods [1]. In fact, it has been verified that with the FRM approach without the concept of don't care bands, the complexity reduction is to about 48% of the complexity yielded by the standard minimax approach. When using the concept of don't care bands, the reduction increases even further to about 35% of the standard one. This results from the fact that in practice we can relax the specifications within the don't care bands, and increase the weighting within the important bands of the filters required by the FRM method. Following this procedure, there are two critical bands that occur at the transition bands of the masking filters, where there is relatively poor cancellation of the two branches in the filter, and the resulting ripple can be significant. In this paper, we introduce the use of the WLS-Chebyshev algorithm for designing the base filter in the FRM method, thus restraining the peaks at the critical bands of the interpolated filter. The result is further reduction in the computational complexity of the resulting filter to approximately 30% of the reference one.

The organization of this paper is as follows: In Sections 2 and 3, we describe the main concepts behind the FRM and WLS-Chebyshev methods, respectively. In Section 4 and 5, we then combine the two methods and describe the whole procedure for designing a lowpass prototype FIR filter with reduced computational complexity.

2. FREQUENCY-RESPONSE MASKING APPROACH

The basic block diagram for the FRM approach can be seen in Figure 1. In this scheme, the so-called interpolated base filter presents

a repetitive frequency spectrum which is processed by the positive masking filter in the upper branch of this realization. Similarly, a complementary version of this repetitive frequency response is operated by the negative masking filter in the lower branch of the realization. In this procedure, both masking filters keep some of the spectrum repetitions which are then added together to compose the desired overall frequency response. The magnitude responses of the filter composing this sequence of operations are depicted in Figure 2, where one can clearly see the resulting filter with very sharp transition band.

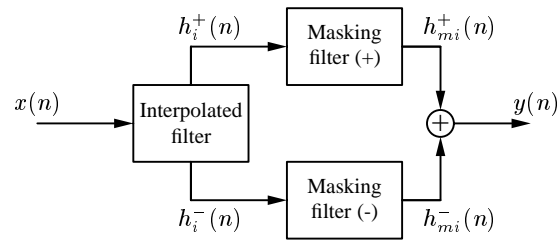


Figure 1: The basic realization of a reduced FIR filter using the frequency-response masking approach.

If the base filter has linear-phase and an even order N , its direct and complementary transfer functions are given by

$$H_i^+(z) = \sum_{i=0}^N h_b(i) z^{-Li} \quad (1)$$

$$H_i^-(z) = z^{-N/2} - \sum_{i=0}^N h_b(i) z^{-Li} \quad (2)$$

respectively, where L is the interpolation factor and $h_b(n)$ is the impulse response of the base filter. From the equations above, we can readily see that

$$|H_i^-(e^{j\omega})| = 1 - |H_i^+(e^{j\omega})| \quad (3)$$

and also that $|H_i^-(e^{j\omega})|$ can be obtained by subtracting $|H_i^+(e^{j\omega})|$ from the signal at the central node in $H_i^+(z)$.

The cutoff frequencies θ and ϕ of the base filter (see Figure 2) depend on L and on the desired band-edge frequencies ω_p and ω_s of the overall filter. The masking filters are simple FIR filters with band-edge frequencies that also depend on L and on the bands of the interpolated filter. Therefore the optimal value of L that minimizes the overall number of multiplications can be obtained by estimating the lengths of all sub-filters for various L and finding the best case scenario empirically.

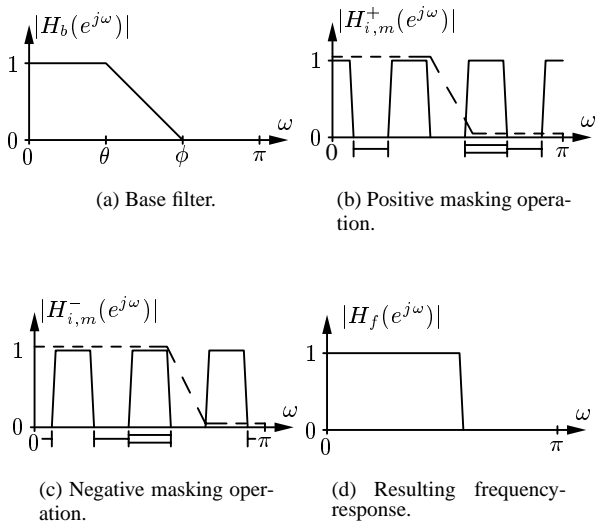


Figure 2: Frequency-response masking approach, showing the don't care bands (single line) and the critical bands (double lines below the frequency axis).

As the frequency responses in each branch depicted in Figure 2 are complementary, the corresponding passband ripples should cancel each other, specially if the two masking filters have approximately the same length. Using the concept of gain margins to determine the specifications for each sub-filter, we can see from the construction of the filter [1] that within the noncritical bands the overall ripple is approximately the sum of the ripple in one of the masking filters (depending on the frequency value) with a second-order term, due to the almost-perfect cancellation of the two branches. This fact must be taken into consideration to determine the specifications for the passband ripple and the stopband attenuation in each subfilter of the FRM design.

For instance, in a design of a low-pass FIR with a desired bandpass ripple of 0.1 dB and minimum stop-band attenuation of 40 dB, the necessary worst-case margin at the noncritical bands is approximately 2.2%, while the worst-case margin at the critical bands are about 50% of the desired overall ripples. Therefore, the weighting at the noncritical bands should be relaxed and the weighting at the critical bands should be increased accordingly in order to accomplish the margin requirements in all frequency bands.

3. WLS-CHEBYSHEV ALGORITHM

Using the FRM algorithm to design filters with large passbands and narrow transition bands, it is verified that the best choices of L lead, in general, to subfilters of similar orders. Also, the base filter after interpolation presents a repetitive (dense) frequency response whereas the masking filters have a more sparse response with ripples having comparatively more energy, as we can see in Figure 3. To explore this fact, we may use a least-squares algorithm to design the base filter, yielding a frequency response with a decaying-peak behavior. However, in order to avoid increasing the minimum attenuation value, we should employ a WLS-Chebyshev method when designing the base filter. In such approach, one is

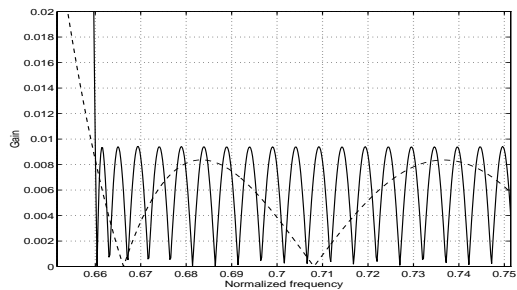


Figure 3: The behavior of the interpolated-base filter (continuous line) and the masking filter (dashed line), at the stopband. If this is one of the critical bands, then the two responses add together and the resulting filter may not satisfy the specification.

able to positively combine the large attenuation characteristic of the Chebyshev (minimax) method with the low stopband energy characteristic of the WLS approximation methods [2]. In fact, the WLS-Chebyshev design scheme yields a filter response with a partially equiripple and partially least-squares-like stopband response.

In [3] a computationally efficient method is proposed for designing WLS-Chebyshev filters. If the base filter has symmetric impulse response with a central coefficient, its magnitude response can be written as

$$|H_b(e^{j\omega})| = \sum_{i=0}^{N/2} h_b(i) \text{trig}(\omega, i) \quad (4)$$

where $\text{trig}(\omega, i)$ is a proper trigonometric function, $h_b(n)$ is the impulse response for the base filter and N is the filter order. The WLS solution in practice minimizes the objective function

$$\epsilon = \sum_n r(\omega_n) E^2(\omega_n) \quad (5)$$

using a dense grid of frequencies ω , where $r(\omega_n)$ is a nonnegative weighting function and $E(\omega)$ gives the amplitude error with respect to the ideal response $\hat{H}(e^{j\omega})$, that is

$$E(\omega) = |H(e^{j\omega})| - |\hat{H}(e^{j\omega})| \quad (6)$$

where, in this equation, $H(e^{j\omega})$ and $\hat{H}(e^{j\omega})$ represent the desired and the obtained response for the base filter or the overall filter. We can use a series of WLS designs to achieve the Chebyshev (minimax) solution by using a variable weighting function at each iteration k , as given by [4]

$$r_{k+1}(\omega_n) = \beta_k(\omega_n) r_k(\omega_n) \quad (7)$$

with

$$\beta_k(\omega_n) = \frac{|E_k(\omega_n)|}{\sum_m r_k(\omega_m) |E_k(\omega_m)|} \quad (8)$$

An accelerated version, however, proposed in [4] upgrades $r_k(\omega_n)$ with the envelope of $\beta_k(\omega_n)$ defined above. This can be determined by searching the peaks of $|E_k(\omega_n)|$ for every ω_n and using a piecewise function joining all these extreme points.

If the updating of the weighting function in equation (7) is made constant for a given frequency interval including J equiripple peaks, the resulting frequency response becomes WLS-like with-in this band and quasi-equiripple in the remaining frequencies [3]. In such case, the modified envelope function is as shown in Figure 4, whereas the typical frequency response of the corresponding WLS-Chebyshev filter is as depicted in Figure 5.

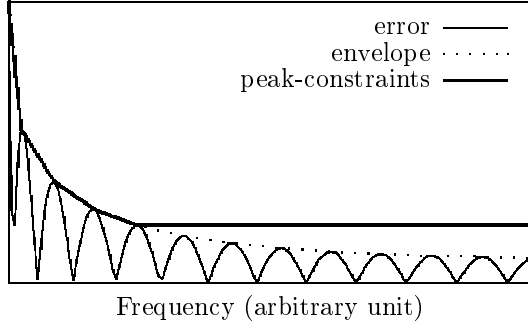


Figure 4: Determining the envelope (dotted line) of the absolute error function to accelerate the minimax convergence, and the modification in [3] (thick line) to design WLS-Chebyshev filters with a constraint for the first $J=5$ peaks.

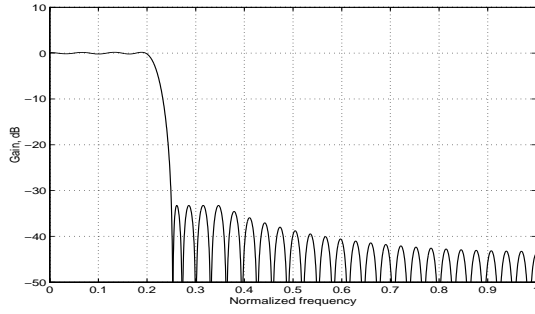


Figure 5: Typical frequency response of a WLS-Chebyshev filter.

4. REDUCED FIR DESIGN

In this section, we describe the use of the WLS-Chebyshev algorithm for improving the frequency response at the critical bands of the overall filter in a FRM design. We start the design by finding the appropriate values of the cutoff frequencies for the subfilters and the value of L which will give the best reduction for the filter. As there is no estimate for the number of the coefficients to be used in a WLS-Chebyshev design, one can estimate the coefficients for the minimax approach, and verify the final response of the filter, reducing the number of coefficients if possible and re-designing the filter. After this, the masking filters can be designed with a minimax approach, employing the concept of don't care bands, adjusting the weights in each band in such a way that the critical bands receive higher weights. The next step is to locate the repetition of the base filter spectrum which is responsible for the

sharp transition of the filter. These frequencies are given by [1]

$$\omega_1 = m \frac{\pi}{L} \quad (9)$$

$$\omega_2 = (m + 1) \frac{\pi}{L} \quad (10)$$

where m is the largest integer such that ω_2 is immediately below the largest cutoff frequency ω_s of the masking filters. These two frequencies, ω_1 and ω_2 , are the centers of the first and second critical bands, respectively. Once these frequencies are found, we can map the masking filter responses over the base filter response, and estimate the resulting error as given in equation (6), with

$$\begin{aligned} |H(e^{j\omega})| &= |H_m^+(e^{j\omega})H_i^+(e^{j\omega}) + H_m^-(e^{j\omega})H_i^-(e^{j\omega})| \\ &= |H_m^+(e^{j\omega})H_i^+(e^{j\omega}) + H_m^-(e^{j\omega})[1 - H_i^+(e^{j\omega})]| \end{aligned} \quad (11)$$

over the interval $\omega \in [\omega_1, \omega_2]$. As we are interested on optimizing the base filter, we can map the frequency responses of the masking filters back to the frequency interval $[0, \pi]$, yielding

$$|H(e^{j\omega})| = |H_m^+(e^{j\omega'})H_b(e^{j\omega}) + H_m^-(e^{j\omega'})[1 - H_b(e^{j\omega})]| \quad (12)$$

with, in this case, $0 \leq \omega \leq \pi$, and

$$\omega' = \omega_1 + (\omega_2 - \omega_1) \frac{\omega}{\pi} \quad (13)$$

if the positive masking filter has cutoff frequencies below the negative masking filter, or

$$\omega' = \omega_2 - (\omega_2 - \omega_1) \frac{\omega}{\pi} \quad (14)$$

if the positive masking filter has cutoff frequencies above the negative masking filter cutoff. This definition of ω' means that depending on which of the two branches is responsible for the last part of the passband, one needs to do a direct or inverse mapping on the frequency, according to equations (13) or (14), respectively. The last step is to determine the peak-constrained frequencies. For this project, we use the first bandstop peak ("side-lobe") of the masking filter. In the frequencies above this peak, it is supposed that the least-squares part of the base filter will cancel the other peaks of the masking filters. Thus, in each iteration, we seek for the first bandstop peak to determine where the envelope function is kept constant. Once the peak-constrained frequencies are known, the optimization algorithm can be applied to design the base filter. In Table 1 we see the design results for various frequencies specifications. Usually, the interpolation factor should be dependent of the sharpness of the transition band, but it can also be different for the two algorithms. By using the same value of L in both algorithms, it is easier to compare the results, because the subfilters will keep the same frequency specifications for both algorithms, thus avoiding the masking filter to operate on different bands of the interpolated base filter.

5. NUMERICAL EXAMPLE

As an example, we show the design of a lowpass reduced FIR filter, with cutoff frequencies of $\omega_p = 0.65\pi$ and $\omega_s = 0.66\pi$, maximum ripple at the passband of 0.2dB and minimum attenuation at the stopband of 40dB. The direct FIR filter implementation using a minimax design will require 382 coefficients, while using a

Table 1: Results obtained by using various frequency specifications. For these designs, the maximum allowable ripple at the passband is 0.2dB and the minimum attenuation is 40dB at the stopband.

Specifications		Minimax		WLS-Chebyshev	
ω_p	ω_s	L	Red. Fact.	L	Red. Fact.
0.178π	0.180π	14	14.74%	14	12.85%
0.240π	0.245π	10	23.72%	10	21.63%
0.32π	0.33π	8	34.91%	8	31.25%
0.65π	0.66π	7	34.82%	7	30.1%

standard FRM minimax with don't care bands the number of coefficients is reduced to 133 for the optimum choice of $L = 7$. By using a quasi-equiripple WLS design [4] on the masking filters, we obtain the frequency-response depicted in Figure 8 (dashed lines). We can then notice from this figure, that by using $J = 5$ equiripple peaks in the WLS-Chebyshev design of the base filter we are able to restrict the critical peaks of the overall design. The overall filter amplitude response is shown in Figure 6, while in Figures 7 and 8 we see all the response at the critical bands.

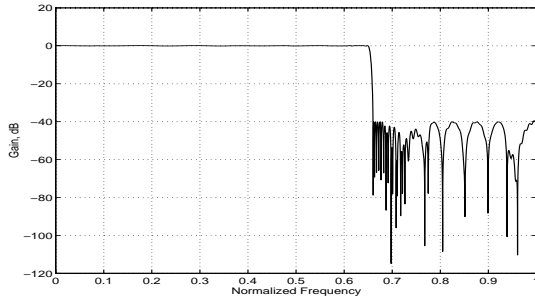


Figure 6: Amplitude response for the example filter.

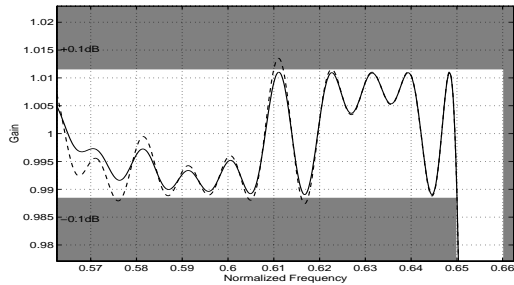


Figure 7: Amplitude response at the first critical band for the example filter (continuous line) and the response of the positive branch (dashed line).

For this design, the result and the comparison between the minimax and the proposed approaches are shown in Table 2. In this table, M denotes the number of coefficients for each of the sub-filters, M_{Tot} is the total number of coefficients (multipliers) on the resulting filter, and the last column is the reduction factor,

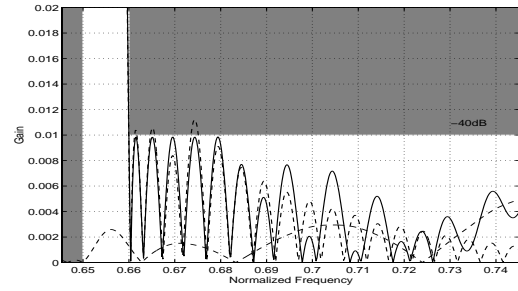


Figure 8: Amplitude response at the second critical band for the example filter (continuous line) and the responses of the two branches of the filter (dashed lines).

given by M_{Tot} divided by the number of coefficients required by the direct implementation.

Table 2: Comparison between the designs using the minimax and the WLS-Chebyshev algorithms.

Algorithm	L	M_b	M_+	M_-	M_{Tot}	Red. Fact.
Minimax	7	65	39	29	133	34.82%
WLS-Chebyshev	7	57	32	26	115	30.1%

6. CONCLUSIONS

We introduced a new design method for FIR digital filters. The proposed method combines the WLS-Chebyshev approach with the frequency-response masking (FRM) method. The main advantage on the WLS-Chebyshev algorithm is the flexibility to work with the weighting function, given any arbitrary error function. We can then design the FRM base filter with a relaxed set of specifications, using the WLS-Chebyshev design. The result is a computationally efficient prototype filter.

7. REFERENCES

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