DESIGN OF COSINE-MODULATED FILTER BANK PROTOTYPE FILTERS USING THE FREQUENCY-RESPONSE MASKING APPROACH

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ABSTRACT

In this paper, we use the frequency-response masking (FRM) approach to design prototype filters for cosine-modulated filter banks in the nearly-perfect reconstruction case. With such approach, it is possible to design a FRM filter with overall order almost equal to the direct-form FIR design, with only slight changes in the values of the inter-carrier and inter-symbol interferences and the attenuation of the bank filters. The result is an efficient design with reduced number of multipliers for the overall structure.

1. INTRODUCTION

The frequency-response masking (FRM) approach is an efficient method for designing linear-phase FIR digital filters with wide passbands and sharp transition bands. With such method, by allowing a small increase of the filter group delay, it is possible to reduce the overall filter complexity (number of arithmetic operations required per output sample) when compared to standard design methods [1]. Since FRM filters are easy to design, it is possible to use them in many practical applications. In fact, it has been verified that the FRM approach can achieve a reduction to about 30% of the number of coefficients required by a minimax FIR filter design realized in direct form [1]. Although the use of the FRM approach increases the filter group delay, in many cases it is possible to design a prototype filter for a cosine-modulated filter bank (CMFB) whose overall order is almost the same for the direct form FIR design. In those cases, the parameters of interest of the transmultiplex (TMUX) system (namely inter-symbol and inter-carrier interferences, and attenuation) are quite close to those in the standard design, the overall delay is not increased, whereas the number of coefficients is reduced. Therefore it is possible to achieve a reduced complexity in a CMFB structure. The organization of this paper is as follows: In Section 2, we describe the main ideas of the FRM approach. In Section 3, we describe the TMUX system. In Section 4, we propose an efficient structure of CMFB, and in Section 5, a design example is included illustrating the advantages of the proposed design method.

2. FREQUENCY-RESPONSE MASKING APPROACH

The basic block diagram for the FRM approach can be seen in Figure 1. In this scheme, the so-called interpolated base filter presents a repetitive frequency spectrum which is processed by the positive masking filter in the upper branch of this realization. Similarly, a complementary version of this repetitive frequency response is operated by the negative masking filter in the lower branch of the realization. In this procedure, both masking filters keep some of the spectrum repetitions within the desired passband which are then added together to compose the desired overall frequency response. The magnitude responses of the filter composing this sequence of operations are depicted in Figure 2, where one can clearly see the resulting filter with very sharp transition band.

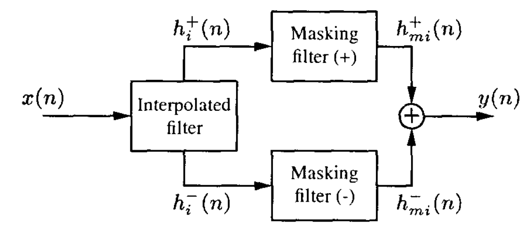

Fig. 1. The basic realization of a reduced FIR filter using the frequency-response masking approach.

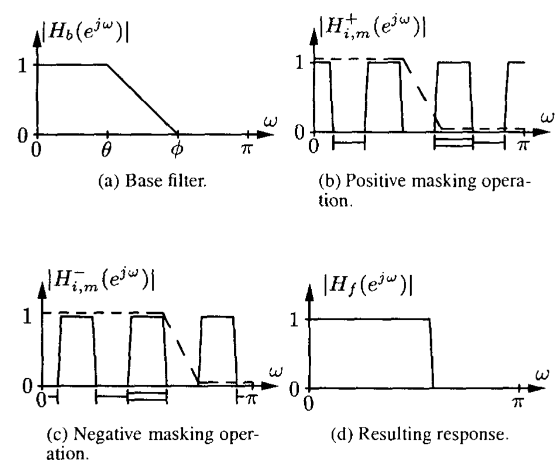

Fig. 2. Frequency-response masking approach, showing the don't care bands (single line) and the critical bands (double lines below the frequency axis).

If the base filter, which will compose the interpolated filter, has linear-phase and an even order N_b, its direct and complementary

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transfer functions are given by

$$H_i^+(z) = \sum_{i=0}^{N_b} h_b(i) z^{-Li} \quad (1)$$

$$H_i^-(z) = z^{-N_b/2} - \sum_{i=0}^{N_b} h_b(i) z^{-Li} \quad (2)$$

respectively, where L is the interpolation factor and $h_b(n)$ is the impulse response of the base filter. From the equations above, we can readily see that

$$|H_i^-(e^{j\omega})| = 1 - |H_i^+(e^{j\omega})| \quad (3)$$

and also that $|H_i^-(e^{j\omega})|$ can be obtained by subtracting $|H_i^+(e^{j\omega})|$ from the signal at the central node in $H_i^+(z)$. The cutoff frequencies θ and ϕ of the base filter (see Figure 2) depend on L and on the desired band-edge frequencies ω_p and ω_s of the overall filter. The masking filters are simple FIR filters with band-edge frequencies that also depend on L and on the bands of the interpolated filter. Therefore the optimal value of L that minimizes the overall number of multiplications can be obtained by estimating the lengths of all sub-filters for various L and finding the best case scenario heuristically. If the transition band is not too sharp when compared to the passband (i.e., for the narrowband design case), then it is possible to discard the lower branch of the FRM filter, reducing further the number of coefficients in the filter. Also, the specifications for the subfilters can be relaxed, since there is no overlap between the frequency-response of the two branches [1]. The narrowband case is common in most of the cases when we are designing CMFBs, but it depends on the roll-off factor and the required attenuation.

3. THE TMUX CONFIGURATION

The transmultiplexer (TMUX) can be implemented with the CMFB in which the signals that come from various sources are interpolated, filtered by synthesis filters and added together to compose a single signal that is transmitted on a single channel C [2],[3]. Once the signal is received, the analysis filters split this signal into M channels, where in each channel output we have an estimated version of the input sources. If the TMUX has perfect-reconstruction (PR), then the output signals are equal to the source signals, whereas if the estimated signal receives small amounts of interference from the other sources, we have the nearly-perfect reconstruction (NPR) case. Figure 3 depicts the block diagram for such system.

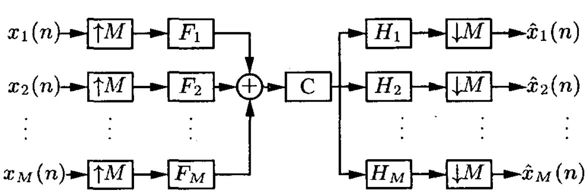


Fig. 3. The block diagram for a M-channel TMUX.

The main advantage by using a CMFB is the fact that only the prototype filter design is needed [2]. Once this filter is designed, the synthesis and the analysis filters can be obtained by modulating the prototype filter with a proper cosine function. The prototype filter of order N_p is of the form

$$H_p(z) = \sum_{n=0}^{N_p} h_p(n) z^{-n}, \quad h_p(N_p - n) = h_p(n) \quad (4)$$

The cutoff frequencies can be determined by using the roll-off factor and the 3dB point of the amplitude response, that must be located approximately at $\omega = \pi/(2M)$. The roll-off factor gives us the stopband edge

$$\omega_s = \frac{(1+\rho)\pi}{2M} \quad (5)$$

The analysis and the synthesis filters are given respectively by

$$h_k(n) = 2h_p(n) \cos\left(\frac{(2k+1)(n - N_p/2)\pi}{2M} + (-1)^k \frac{\pi}{4}\right) \quad (6)$$

and

$$f_k(n) = 2h_p(n) \cos\left(\frac{(2k+1)(n - N_p/2)\pi}{2M} - (-1)^k \frac{\pi}{4}\right) \quad (7)$$

for $k = 0, 1, \ldots, M-1$. The transfer matrix for the filter bank is given by

$$[\mathbf{T}(z^M)]_{ab} = \sum_{k=0}^{M-1} H_a(ze^{-j2\pi k/M}) F_b(ze^{-j2\pi k/M}) \quad (8)$$

which represents the various input to output relationships on the TMUX system. By taking some particular transfer functions based on the transfer function matrix (8), one can define the following functions

$$T_0(z) = \frac{1}{M} \sum_{k=0}^{M-1} F_k(z) H_k(z) \quad (9)$$

$$T_i(z) = \frac{1}{M} \sum_{k=0}^{M-1} F_k(z) H_k(ze^{-j2\pi i/M}) \quad (10)$$

for $i = 1, \ldots, M-1$, and the optimization problem is to find the coefficients of $H_p(z)$ to minimize

$$E_2 = \int_{\omega_s}^{\pi} |H_p(e^{j\omega})| d\omega \quad (11)$$

subject to

$$1 - \delta_1 \leq |T_0(e^{j\omega})| \leq 1 + \delta_1 \quad (12)$$

and

$$|T_i(e^{j\omega})| \leq \delta_2 \quad \omega \in [0, \pi], \text{ for } i = 1, \ldots, M-1 \quad (13)$$

in which the parameters δ_1 and δ_2 are constraints that define the maximum allowable amplitude distortion and aliasing distortion, respectively, of the bank filters. The optimization problem becomes non linear because of the two constraints given in Eqs. (12) and (13), but it can be simplified by using one of the constraints and verifying if the other constraint is achieved. For instance, if δ_1 becomes zero and δ_2 (measured in dBs) becomes infinitely negative, then the PR is achieved. For the NPR case, it is possible to use prototype filter designs that approximate the two constraints above. The inter-symbol interference (ISI) and the inter-carrier interference (ICI) can be estimated by using the following expressions [3]

$$\mathrm{ISI} = \max_k\left(\sum_n(\delta(n) - t_k(n))^2\right) \tag{14}$$

$$\mathrm{ICI} = \max_k\left(\sum_{l=0,k\neq l}^{M-1} |[\mathbf{T}(e^{j\omega})]_{kl}|^2\right) \tag{15}$$

where $\delta(n)$ is the ideal impulse, $t_k(n)$ is the impulse response for the kth channel output, and the term $[\mathbf{T}(e^{j\omega})]_{kl}$ is the crosstalk which can be obtained from Eq. (8).

4. FRM DESIGN FOR THE PROTOTYPE FILTER

The FRM filter can be viewed as depicted in Figure 4. In this figure, $H_{b1}(z)$ and $H_{b2}(z)$ are the base filter and its complementary, while $G_1(z)$ and $G_2(z)$ are the masking filters.

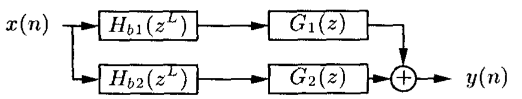


Fig. 4. The positive (upper) and the negative (lower) branches of the FRM filter.

By using only the positive branch of the FRM structure as the prototype filter for the CMFB, the transfer function for the analysis filters becomes

$$H_m(z) = \sum_{n=0}^{N} c_{m,n}(h_{b1}^I * g_1)(n)z^{-n} \tag{16}$$

where $c_{m,n}$ is the cosine function as it appears in Eq. (6), the term $(h_{b1}^I * g_1)(n)$ denotes the convolution between the interpolated base filter and the positive masking filter responses, and N is the overall order of the FRM filter. Thus, the key point is to find an efficient structure that evaluates the convolution in Eq. (16), by taking into consideration the proper cosine functions for each sample. For $L = \frac{M}{K}, K = 1, 2, \ldots$, and after some manipulations, Eq. (16) can be written as

$$H_m(z) = \sum_{q=0}^{Q-1}\left\{z^{-Lq}H_{b1_q}(-z^{2M})\sum_{j=0}^{2M-1} c_{m,j+Lq}z^{-j}E_j(-z^{2M})\right\} \tag{17}$$

where $H_{b1_q}(z)$ and $E_j(z)$ are the polyphase decompositions of the base filter and of the positive masking filter, respectively, and $Q = \frac{2M}{L}$ is the number of polyphase decompositions required for the base filter. This result leads us to a structure as depicted in Figure 5. The same can be done for the negative branch of the FRM, and then adding the responses of the two branches just before the modulating stage. As will be demonstrated on a future work, this structure can be conveniently reduced and generalized for several cases of L and M.

As we can see from Figure 5, the base filter will have $M_b = K_a Q$ coefficients and the masking filter will have $M_m = 2K_b M$ coefficients in order to perform the polyphase decompositions. The values of K_a and K_b can be chosen to lead to an overall filter

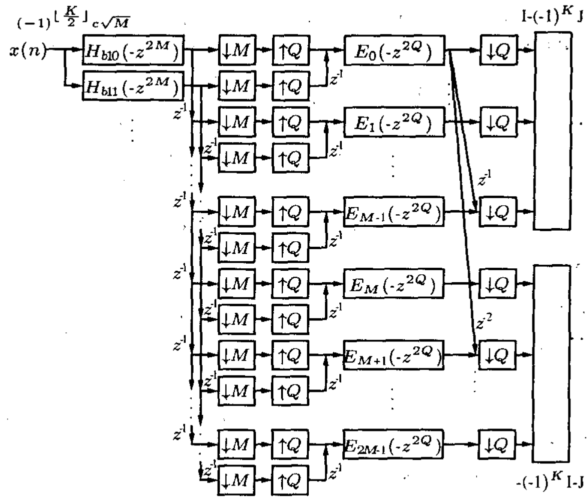


Fig. 5. The realization of the FRM in a CMFB structure that precedes the DCT-IV operation.

with $M_{ov} = 2KM$ coefficients which is the same number required by the standard CMFB design. In such cases, it is easier to compare the performances of the FRM-CMFB and the standard-CMFB. In the design of the FRM-CMFB presented in the next section, we aimed a reduction scenario leading to these values. It is worth mentioning that in order to solve the optimization problem for the NPR case, it is possible to employ, for example, the WLS-Chebyshev algorithm [4], constraining the overall response of the FRM filter as desired.

5. NUMERICAL EXAMPLE

In this section, we present a TMUX system design with $M = 32$ channels, using $\rho = 1$ and δ_1 less than 0.06. For this example, $K = 5$ should be sufficient to perform the design. Calculating the frequencies for the design of the prototype filter, we obtain the following values: $\omega_p = 0.005469\pi$ and $\omega_s = 0.03125\pi$, leading to $\omega_{3dB} \approx \frac{\pi}{2M} = 0.0156\pi$. The order for the prototype filter is: $N_p = 2KM - 1 = 319$ (320 coefficients). Using a standard minimax FIR design for the prototype filter, we obtain the results presented at the first line of Table 1. By using a FRM design, we see that the best reduction scenario which leads to an order of 320 is as given in Table 2. The comparison is presented in Table 1.

Table 1. Comparison between the CMFBs performances using a direct-form prototype filter design and a FRM design, for the numerical example.

Method	N_p	N.C.	δ_1	δ_2	A_r	ISI	ICI
Direct	319	320	0.006	-66dB	116dB	-100dB	-63dB
FRM	320	70	0.002	-66dB	105dB	-114dB	-61dB

As we can see from Table 1, using a FRM design will lead to a reduced number of coefficients (N.C., third column of the table), whereas the values of ISI and ICI are approximately the same when compared to the direct-form FIR implementation. Also, the value of δ_2 has remained the same, whereas the value of δ_1 has been decreased. The use of FRM has increased the value of the maximum attenuation A_r of the channel at the stopband, consequently increasing slightly the value of the ICI. If we wish to reduce these values, it is possible to implement an FRM design by increasing its order, and even in this case, the number of coefficients needed may still be lower than the direct form implementation.

Table 2. The best reduction choices for the FRM filter with overall order approximately equal to the direct form, for the numerical example.

L	N_b	N_+	N_-	N_{ov}	Num. Coef	Red. fact
8	36	32	-	320	70	21.9%

In Figures 6 and 7, we see the frequency-response behavior of the prototype filter and the bank filters, while in Figure 8 we see some of the T_i functions that are responsible for the values of δ_1 and δ_2.

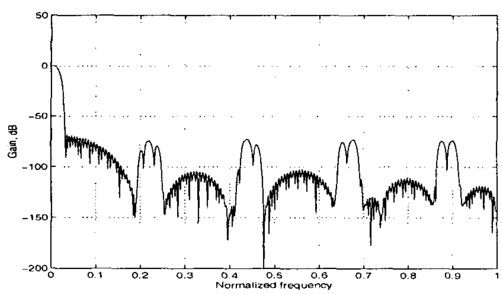


Fig. 6. The magnitude response for the prototype filter in the FRM approach.

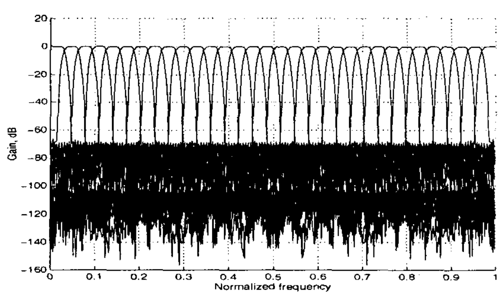
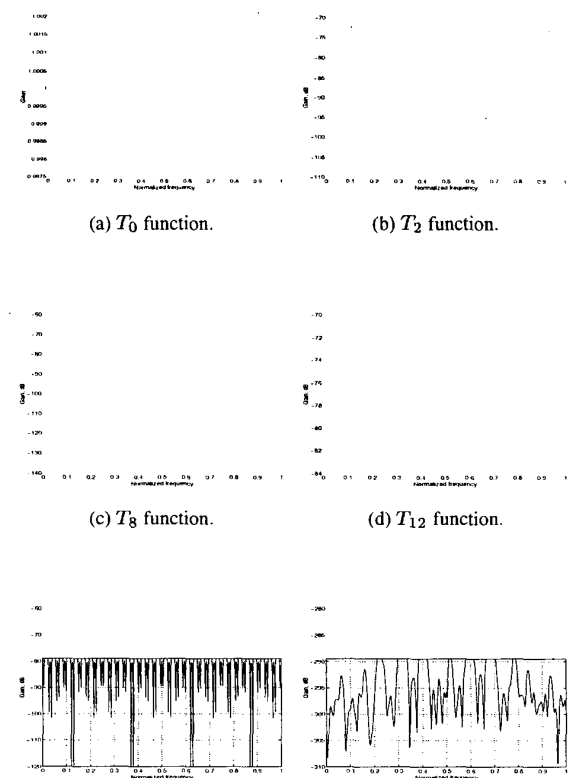


Fig. 7. The magnitude response for the various synthesis/analysis filters.

6. CONCLUDING REMARKS

In this paper, we used the frequency-response (FRM) approach for designing the prototype filter in a cosine-modulated filter bank. By viewing the FRM filter as a multirate system, it was possible to derive an efficient realization for the filter bank when the number of channels M is a multiple of the interpolation factor L of the FRM



(a) T_0 function. (b) T_2 function.

(c) T_8 function. (d) T_{12} function.

(e) T_{15} function. (f) T_{17} function.

Fig. 8. Some of the T_i functions that contribute to the deviation errors δ_1 and δ_2.

filter. The time delay of the resulting TMUX was not affected, and only slight changes were observed in the parameters of interest of the filter bank. Thus, it was possible to reduce the number of coefficient in the TMUX, while maintaining the polyphase decomposition structure.

7. REFERENCES

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