EFFICIENT IMPLEMENTATION FOR COSINE-MODULATED FILTER BANKS USING THE FREQUENCY-RESPONSE MASKING APPROACH

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ABSTRACT

Frequency-response masking (FRM) approach is a very efficient method for designing linear-phase FIR digital filters with general passbands and sharp transition bands. The FRM approach has recently been adapted to design cosine-modulated filter banks (CMFB). The resulting structure is a two-stage filter formed by the polyphase decomposition of the base filter in cascade with the decomposition of the masking filter. The present paper discusses the implementation of the FRM-CMFB using a butterfly-type structure proposed by Malvar to implement the masking filter polyphase decomposition. The result is an even more efficient structure in terms of computational complexity. The issue of perfect reconstruction (PR) in this new structure is also addressed, and it is shown how the PR of the overall filter depends on the characteristics of the FRM base and masking filters.

1. INTRODUCTION

Frequency-response masking (FRM) approach is a very efficient method for designing linear-phase FIR digital filters. With such method, it is possible to reduce the overall filter complexity (number of arithmetic operations required per output sample) of the filter, when compared to standard design methods. Since FRM filters are easy to design, they have been studied for many practical applications. In addition, recent results show that the FRM framework can be generalized to accommodate different structures and approximation methods [2, 3, 4].

The cosine-modulated filter banks (CMFB) are very popular in applications requiring large number of subbands due to their easy design and efficient implementation, in terms of computational complexity [5, 6, 7]. Recently, the frequency response masking approach was used to design CMFB, leading to a two-stage decomposition filter, namely the decomposition of the base filter cascaded with the decompositions of the masking filter. The polyphase decomposition analysis for this combined structure can be found in [7]. As one can observe the structure, it is possible to use butterflies structures for the masking filter, by replacing the polyphase decomposition stage of the masking filter, using the butterfly structure instead. While one can guarantee the PR property of the masking filter, the overall filter characteristics will depend on the base filter. In this paper, we review some of the FRM and CMFB concepts, and present the butterfly-based structures for the TMUX case. With this new structure we achieve an even higher degree of computtional savings.

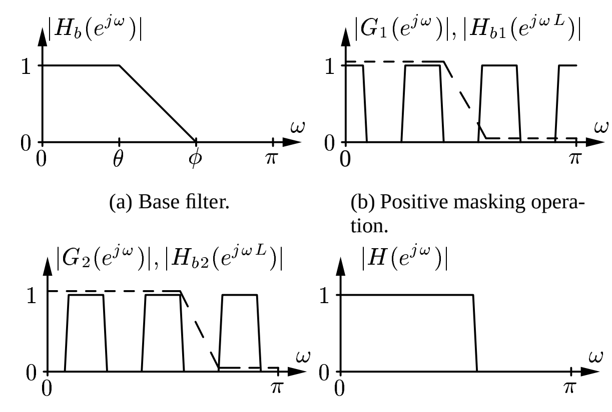
The organization of this paper is as follows: In Section 2, we describe the main ideas of the FRM approach. In Section 3, we describe the TMUX system. In Section 4, we derive the FRM-CMFB general structure. The butterfly-based structure for the FRM-CMFB is presented in Section 5, leading to more efficient structure with respect to computational complexity. In Section 6, we present a design procedure with the proposed method.

2. FREQUENCY-RESPONSE MASKING APPROACH

The FRM approach is described by

$$H(z) = H_{b1}(z^L)G_1(z) + H_{b2}(z^L)G_2(z) \tag{1}$$

where, if $H_b(z)$ is a base filter and L the interpolation factor, then the so-called interpolated base filter $H_{b1}(z^L)$ presents a repetitive frequency spectrum whose output signal is processed by the positive masking filter $G_1(z)$. Similarly, a complementary version of this repetitive frequency response, $H_{b2}(z^L)$, is in cascade with the negative masking filter $G_2(z)$. In this procedure, both masking filters keep some of the spectrum repetitions within the desired passband which are then added together to compose the desired overall frequency response. The magnitude responses of the filter composing this sequence of operations are depicted in Figure 1, where one can clearly see that the resulting filter has a very sharp transition band.



(a) Base filter. (b) Positive masking operation. (c) Negative masking operation. (d) Resulting response.

Figure 1: The combination of the frequency responses in the upper and lower branches generates the FRM filter with very narrow transition band.

If the base filter has linear-phase and an even order N_b, its direct and complementary transfer functions, interpolated by a factor of L, are given by

$$H_{b1}(z^L) = \sum_{i=0}^{N_b} h_b(i) z^{-Li} \tag{2}$$

$$H_{b2}(z^L) = z^{-N_b/2} - \sum_{i=0}^{N_b} h_b(i) z^{-Li} \tag{3}$$

respectively, where $h_b(n)$ is the impulse response of the base filter.

The cutoff frequencies θ and ϕ of the base filter (see Figure 1) depend on L and on the desired bandedge frequencies ω_p and ω_s of the overall filter. The masking filters are simple FIR filters whose bandedge frequencies also depend on L and on the bands of the interpolated filter. Therefore the value of L that minimizes the overall number of multiplications can be obtained by estimating the lengths of all sub-filters for various L, and finding the best case scenario heuristically. Then, it is possible to optimize the responses of the sub-filters by using a variety of algorithms [3, 4]. If the transition band is not too sharp when compared to the passband (i.e., for the narrowband design case), then it is possible to discard the lower branch of the FRM filter, further reducing the number of coefficients in the overall filter. Also, the specifications of the subfilters can be relaxed, since there is no overlap between the frequency responses of the two branches [1]. The narrowband case is commonly found when we are designing CMFBs, depending on the required roll-off factor and the attenuation level.

3. THE TMUX CONFIGURATION

The transmultiplexer (TMUX) is an application of filter banks in which the signals coming from various sources are interpolated, filtered by a synthesis bank, and added together to compose a single signal for transmission over a given channel C [5, 8]. At the receiver, the analysis filters split the transmitted signal back into M channels, where each output corresponds to one original input source. If the TMUX has perfect reconstruction (PR), then the output signals are equal to the source signals when the channel model is a pure delay, whereas if the estimated signals receive small interference from the other sources, we have the nearly-perfect reconstruction (NPR) case. Figure 2 depicts the block diagram for such system.

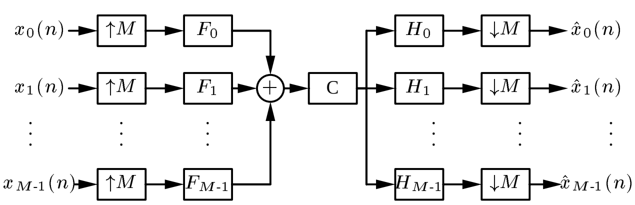


Figure 2: The block diagram for a M-channel TMUX.

The main advantage of using a CMFB is the fact that only one prototype filter design is required [5, 9], since the synthesis and analysis filters are obtained by modulating this filter with a proper cosine function. For the prototype filter, the 3 dB attenuation point of the magnitude response is located at approximately

$$\omega_{3\text{dB}} \approx \pi/(2M) \tag{4}$$

and the stopband edge is determined by the roll-off factor, ρ, as

$$\omega_s = \frac{(1+\rho)\pi}{2M} \tag{5}$$

If the prototype filter of order N_p has a transfer function

$$H_p(z) = \sum_{n=0}^{N_p} h_p(n) z^{-n} \tag{6}$$

then the impulse responses of the analysis and the synthesis filters are given respectively by

$$h_m(n) = 2h_p(n)\cos\left[\frac{(2m+1)(n-N_p/2)\pi}{2M} + (-1)^m\frac{\pi}{4}\right] \tag{7}$$

$$f_m(n) = 2h_p(n)\cos\left[\frac{(2m+1)(n-N_p/2)\pi}{2M} - (-1)^m\frac{\pi}{4}\right] \tag{8}$$

for $m = 0, 1, \ldots, (M-1)$, and $n = 0, 1, \ldots, N_p$.

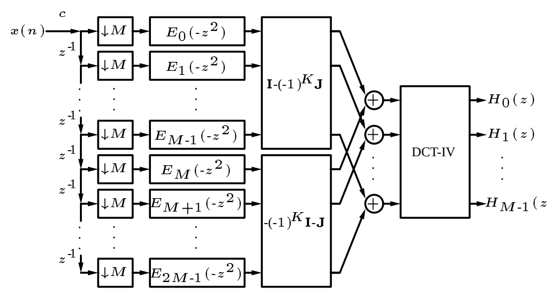


Figure 3: The efficient CMFB structure.

If the prototype filter has $2KM$ coefficients, then the efficient structure for the filter bank, depicted in Figure 3, is achieved. From this figure, it is easy to infer that the CMFB realization requires only (N_p+1) multiplications for the polyphase components, one multiplication by c, the modulating matrices consisting of $[\mathbf{I} - (-1)^K\mathbf{J}]$, $[-(-1)^K\mathbf{I} - \mathbf{J}]$, and the DCT-IV. The overall number of multiplications (not taking into consideration any symmetry of coefficients) is then

$$\mathcal{M} = 2KM + \alpha \tag{9}$$

where α is associated to the multiplication by the constant c and the modulating matrices, which can be implemented using fast algorithms. The computational complexity (number of multiplications to generate one output sample) for the structure of Figure 3, considering the decimation, is then

$$\mathcal{C} = 2K + \beta \tag{10}$$

where β is related to the constant c and the modulating matrices, just like α. The polyphase decomposition turns the value of $\mathcal{C}$ small, even if the number of channels M is large, demonstrating the efficiency of the structure.

4. THE FRM-CMFB STRUCTURE

From the analysis of the FRM-CMFB structure, we see that it is possible to derive an efficient structure for the case where the interpolator factor is written as [7]

$$L = 2K_aM + \frac{M}{K_b} \tag{11}$$

with K_a a nonnegative integer and K_b a positive integer. In this case, the transfer tunction of the m-th analysis filter is given by

$$H_m(z) = \sum_{i=0}^{N_b} \left[h_b(i) z^{-Li} \sum_{n=0}^{N_m} c_{m,[n+(2K_a M + \frac{M}{K_b})i]} g_1(n) z^{-n} \right] \tag{12}$$

and then, by using $Q = 2K_b$, $i = kQ + q$, and $(N_b + 1) = QK_c$, where Q and N_b are the number of polyphase decompositions and the order of the base filter, respectively, we have, after some manipulations

$$H_m(z) = \sum_{q=0}^{Q-1} \sum_{k=0}^{K_c-1} \left[h_b(kQ+q) z^{-L(kQ+q)} \right. \\ \left. \times (-1)^{(k+K_a q)} \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K_b}q)} z^{-j} E'_j(-z^{2M}) \right] \tag{13}$$

By defining the modified polyphase components of the interpolated base filter as

$$H'_{b1q}(z) = \sum_{k=0}^{K_c-1} (-1)^{K_a q} h_b(kQ+q) z^{-k} \tag{14}$$

with $q = 0, 1, \dots, (Q-1)$, Eq. (13) can be rewritten as

$$H_m(z) = \sum_{q=0}^{Q-1} \left[z^{-Lq} H'_{b1q}(-z^{LQ}) \right. \\ \left. \times \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K_b}q)} z^{-j} E'_j(-z^{2M}) \right] \tag{15}$$

and we derive the structure depicted in Figure 4, with K equivalent to the CMFB case.

The values of K_a and K_b can be chosen such that the overall filter has the same number of coefficients required by the standard CMFB design. In such cases, it is easier to compare the structures of the FRM-CMFB and the standard CMFB.

5. THE FRM-CMFB WITH BUTTERFLY STRUCTURES

If the prototype filter has symmetric impulse response, then it is possible to use the butterfly-based structures proposed by Malvar [9] depicted in Figure 5. The resulting filter bank has perfect reconstruction. If it is desired to design an NPR filter bank, a modified structure can be used instead [6, 8]. In the case of Figure 5, the D_k stages of butterflies can be viewed as matrices with nonzero elements only at the main diagonals, thus reducing the computational complexity to about 25% of the value obtained with the polyphase decomposition case.

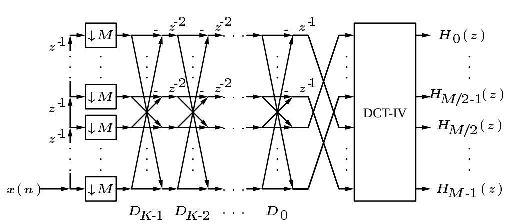


Figure 5: Improved structure for the CMFB using butterfly for the PR case.

Thus, it is also possible to use a butterfly structure for the general case of L, in the FRM-CMFB scheme as depicted in Figure 7. In the case of $L = M$, since the butterfly-based structure requires only decimation by M, the resulting structure will not require the Q multiple-signal filtering stage. This particular but very important case is depicted in Figure 6.

For the general case of $L = 2K_a M + \frac{M}{K_b}$, the computational complexity is reduced due to the butterfly structure, that has fast implementation, and due to the fact that the interpolating factor $Q = K_b$ is half of the value for the polyphase structure case. Now, however, the modified polyphase components of the base filter are given by

$$H''_{b1q}(z) = \sum_{k=0}^{K_c-1} (-1)^{K_a(kQ+q)} h_b(kQ+q) z^{-k} \tag{16}$$

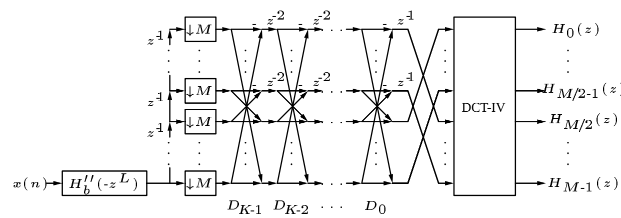


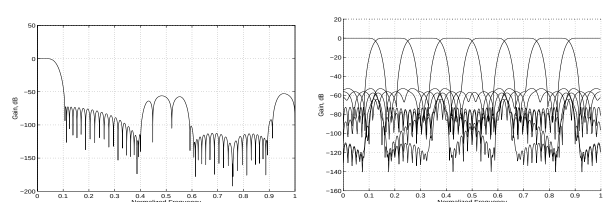
Figure 6: Improved structure using butterfly for the case when $L = M$.

6. EXAMPLE

In this section, we illustrate the ideas above, by designing a filter bank with $M = 8$ channels, using only the upper branch of the FRM and the butterfly structure for the masking filter. Selecting $L = 4$ and $K = 8$ will lead us to the prototype filter with magnitude response depicted in Figure 8a. The results for the FRM prototype filter are shown in Table 1.

Table 1: Results for the FRM prototype filter using the butterfly structure.

L	M_b	M_+	M_-	M_{Tot}
4	29	16	-	128



(a) Prototype filter. (b) Bank filter.

Figure 8: Magnitude responses for the design example.

For this design, $\omega_{3\text{dB}} = 0.0624\pi \approx \frac{\pi}{16}$, and we have $Q = 2$ polyphase decompositions for the base filter. The results of the filter bank are depicted in Figure 8b, and in Table 2 we show the values of the inter-symbol and inter-carrier interferences for this example.

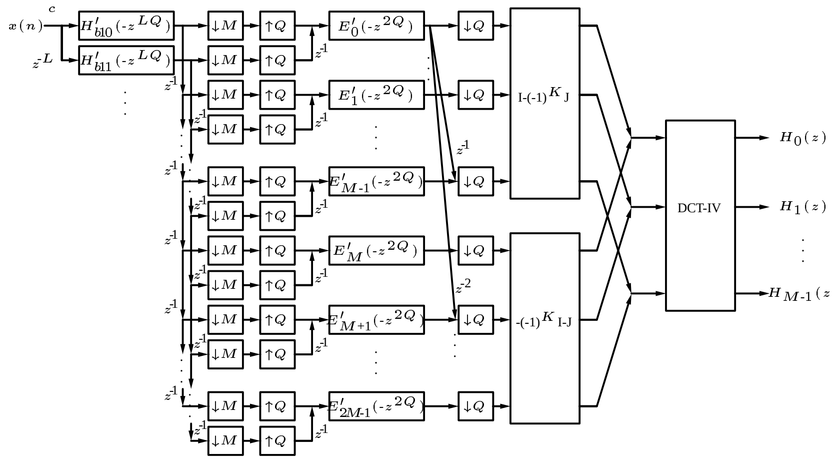


Figure 4: CMFB structure using FRM for the general case of $L = 2K_a M + \frac{M}{K_b}$.

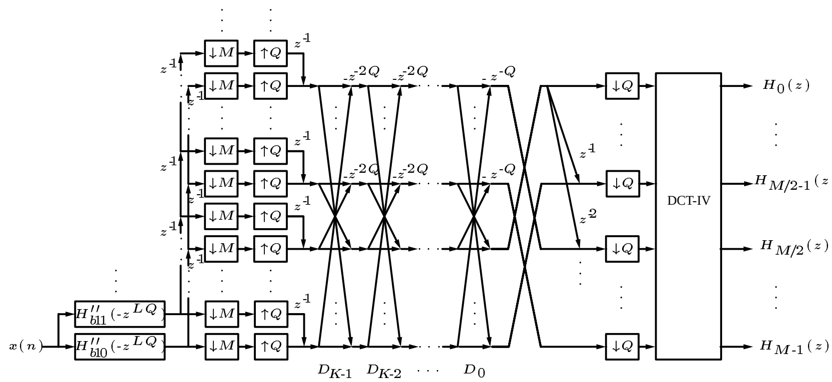


Figure 7: Improved general structure using butterflies.

Table 2: Filter bank results.

ISI	ICI	A_r
-51.6 dB	-62.6 dB	-51.2 dB

7. CONCLUSION

In this paper, it was shown how the frequency-response masking (FRM) approach can be applied for designing the prototype filter in cosine-modulated filter banks. Moreover it was possible to use the butterfly-based structure for the masking filter, since the polyphase decomposition for the masking filter is analogous to the CMFB case. It is possible then to take the full advantage of using the butterfly structure to further reduce the complexity of the FRM-CMFB filter.

8. REFERENCES

[1] Y. C. Lim, "Frequency-response masking approach for the synthesis of sharp linear phase digital filters," *IEEE Trans. Circuits and Systems*, vol. CAS-33, pp. 357–364, Apr. 1986.

[2] Y. C. Lim and Y. Lian, "Frequency-response masking approach for digital filter design: Complexity reduction via masking filter factorization," *IEEE Trans. Circuits and Systems II, Analog and Digital Signal Processing*, vol. 41, pp. 518–525, Aug. 1994.

[3] P. S. R. Diniz and S. L. Netto, "On WLS-Chebyshev FIR Digital Filters", *Journal of Circuits, Systems and Computers*, vol. 9, pp. 155–168, 1999.

[4] L. C. R. de Barcellos, S. L. Netto, and P. S. R. Diniz, "Design of FIR filters combining the frequency-response masking and the WLS-Chebyshev approaches", *Proc. IEEE Int. Symp. Circuits and Systems*, vol. II, pp. 613–616, Sydney, Australia, May 2001.

[5] T. Saramäki, "A generalized class of cosine-modulated filter banks," *Proc. TICSP Workshop Transforms and Filter Banks*, pp. 336–365, Tampere, Finland, June 1998.

[6] J. Alhava and A. Viholainen, "Implementation of nearly perfect-reconstruction cosine-modulated filter banks", *Proc. FinSig*, pp. 222–226, Oulu, Finland, 1999.

[7] P. S. R. Diniz, L. C. R. de Barcellos, and S. L. Netto, "Design of cosine-modulated filter bank prototype filters using the frequency-response masking approach", *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, vol. VI, pp. SPTM-P4.6 1–4, Salt Lake City, UT, May 2001.

[8] A. Viholainen, T. Saramäki, and M. Renfors, "Nearly-perfect reconstruction cosine-modulated filter bank design for VDSL modems", *Proc. IEEE Int. Conf. Electronics, Circuits and Systems*, Paphos, Greece, pp. 373–376, Sept. 1999.

[9] H. S. Malvar, *Signal Processing with Lapped Transforms*, Artech House, Boston:MA, 1991.