Efficient Design of CMFBs Using Two-Stage Frequency-Response Masking

Luiz C. R. de Barcellos, Sergio L. Netto, and Paulo S. R. Diniz
Laboratório de Processamento de Sinais
Universidade Federal do Rio de Janeiro

Abstract— **A new method for designing narrowband cosine-modulated filter banks (CMFBs) is proposed based on the frequency-response masking (FRM) approach with masking filter decomposition. The resulting structure, the so-called FRM2-CMFB, presents a reduced computational complexity, and allows one to design filter banks with extremely high number of bands, where standard filter design methods fail to converge. Examples included, illustrate that the total number of coefficients of the CMFB prototype filter can be reduced to about 60% of the original FRM-CMFB structure, which does not use masking filter decomposition.**

I. Introduction

The cosine-modulated filter banks (CMFBs) are very popular in applications requiring large number of subbands due to their easy design (based solely on a single prototype filter) and computationally efficient implementation [3], [4], [5].

The frequency-response masking (FRM) approach is an efficient method for designing linear-phase FIR digital filters with general passbands and sharp transition bands. With such method, by allowing a small increase in the overall filter order, it is possible to reduce the number of coefficients to about 30% of the number required by a minimax FIR direct-form filter [1].

In this paper, we analyze the use of the FRM approach to design the prototype filter of a CMFB. The narrowband-CMFB case is considered where standard minimax method fails to converge and even the FRM-CMFB [5] structure presents high computational complexity. A new structure is then introduced, the so-called FRM2-CMFB, where the initial FRM masking filter is decomposed into a second-stage interpolated base filter and corresponding masking filters. The result is further reduction in the computational complexity of the CMFB implementation, when compared to the FRM-CMFB, and the possibility of designing filter banks with extremely high number of bands.

The organization of this paper is as follows: In Sections 2 and 3, we describe the basic concepts behind the CMFB and FRM methods, respectively. In Section 4, the FRM-CMFB structure presented in [5] is revised, and a narrowband CMFB design is included. In Section 5, this example is used as motivation to introduce the FRM2-CMFB structure, where a two-stage FRM is used to design the CMFB prototype filter. Examples are included illustrating the results achieved with the proposed method.

II. The CMFB Structure

CMFBs are a commonly used tool in signal processing applications [3], [6], [7], [8]. The main advantages of CMFBs include its simple design, as only one prototype filter is required, and its computationally efficient implementation.

Programa de Engenharia Elétrica-COPPE/EE/UFRJ, PO Box 68504, Rio de Janeiro, RJ, 21945-970, BRAZIL. E-mails: {barcellos, sergioln, diniz}@lps.ufrj.br

The CMFB prototype filter is characterized by a 3-dB attenuation point and the stopband edge frequency given by

$$\omega_{3\text{dB}} \approx \pi/(2M); \quad \omega_s = \frac{(1+\rho)\pi}{2M} \tag{1}$$

where ρ is the so-called roll-off factor that controls the amount of overlapping between adjacent bands. If the prototype filter has order N_p and transfer function

$$H_p(z) = \sum_{n=0}^{N_p} h_p(n) z^{-n} \tag{2}$$

then the impulse responses of the analysis and the synthesis filters are given by

$$h_m(n) = 2h_p(n)\cos\left[\frac{(2m+1)(n-N_p/2)\pi}{2M} + (-1)^m\frac{\pi}{4}\right] \tag{3}$$

$$f_m(n) = 2h_p(n)\cos\left[\frac{(2m+1)(n-N_p/2)\pi}{2M} - (-1)^m\frac{\pi}{4}\right] \tag{4}$$

respectively, for $m = 0, 1, \ldots, (M-1)$, and $n = 0, 1, \ldots, N_p$.

If the CMFB prototype filter has $(N_p+1) = 2KM$ coefficients, then the $2M$-polyphase decomposition on $H_p(z)$ can be performed such that

$$H_p(z) = \sum_{j=0}^{2M-1} z^{-j} E_j(z^{2M}) \tag{5}$$

with

$$E_j(z) = \sum_{k=0}^{K-1} h_p(2kM+j) z^{-k} \tag{6}$$

for $j = 1, \ldots, 2M$.

After some standard algebraic manipulations [7], the analysis filters can then be written as[1]

$$\begin{aligned} H_m(z) &= \sum_{j=0}^{2M-1}\left[c_{m,j} z^{-j} \sum_{k=0}^{K-1} (-1)^k h_p(2kM+j) z^{-2kM}\right] \\ &= \sum_{j=0}^{2M-1} c_{m,j} z^{-j} E_j(-z^{2M}) \end{aligned} \tag{7}$$

for $m = 0, 1, \ldots, (M-1)$. Based on such description, the analysis filter bank can be efficiently implemented as given in Figure 1, where $\mathbf{I}$ is the identity matrix and $\mathbf{J}$ is the reverse identity matrix.

[1] An analogous decomposition follows for the synthesis filters.

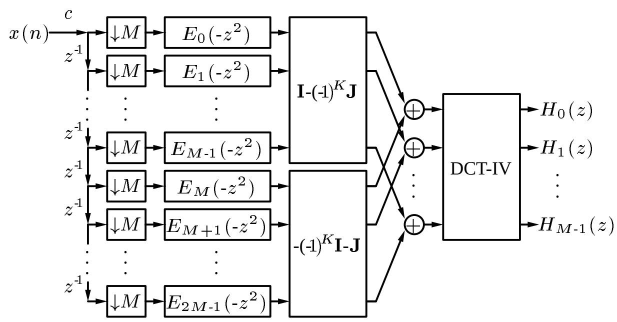


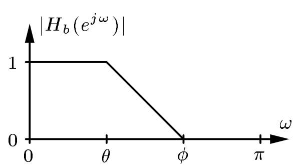
Fig. 1. Efficient CMFB structure.

III. The FRM Method

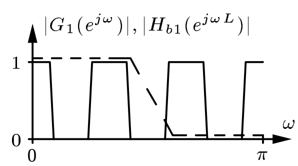
In the FRM technique, a lowpass base filter is interpolated by a factor of L, $H_{b1}(z^L)$, what generates a repetitive frequency spectrum whose output signal is processed by the so-called positive masking filter, $G_1(z)$. Similarly, the complement of this repetitive frequency response, $H_{b2}(z^L)$, is cascaded by the negative masking filter, $G_2(z)$. In such scheme, both masking filters must keep the desired spectrum repetitions within the overall passband, while eliminating the undesired spectrum repetitions. The output of both masking filters are then added together to form the desired overall response, such that

$$H(z) = H_{b1}(z^L)G_1(z) + H_{b2}(z^L)G_2(z) \tag{8}$$

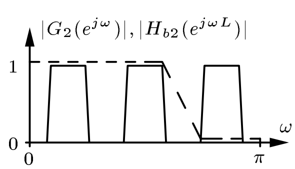
as illustrated in Figure 2, where one can clearly see the sharp transition nature of the resulting filter.



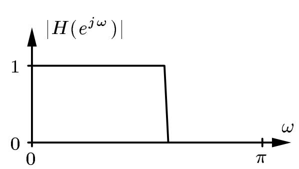
(a) Base filter.



(b) Positive masking filtering.



(c) Negative masking filtering.



(d) Overall FRM response.

Fig. 2. FRM operation: The combination of the frequency responses in the direct and complementary FRM branches generates a filter with general passband and narrow transition band.

If the desired filter is narrowband, then it is possible to eliminate the FRM complementary branch, further reducing the number of coefficients in the resulting filter [1].

IV. The FRM-CMFB Structure

Let us consider the FRM-CMFB structure where the FRM approach is used to design the prototype filter of a CMFB. Assume also that we are mainly interested on the narrowband filter case, where the complementary branch is absent from the FRM structure[2]. In such case, the transfer functions for the analysis filters are given by

$$H_m(z) = \sum_{n=0}^{N} c_{m,n}(h_{b1}^I * g_1)(n)z^{-n} \tag{9}$$

where the term $(h_{b1}^I * g_1)(n)$ denotes the convolution between the interpolated base filter and the positive masking filter responses, and N is the overall order of the FRM filter. The key point in the FRM-CMFB structure is to find out how to obtain a computationally efficient polyphase decomposition of this convolution operation.

Assuming that $H_{b1}(z)$ and $G_1(z)$ have orders N_b and N_m, respectively, and using the definition of convolution, Eq. (9) can be rewritten as

$$H_m(z) = \sum_{i=0}^{N_b}\left[h_b(i)z^{-Li}\sum_{n=0}^{N_m} c_{m,(n+Li)}g_1(n)z^{-n}\right] \tag{10}$$

If the interpolation factor can be written as [5]

$$L = 2K_aM + \frac{M}{K_b} \tag{11}$$

where $K_a \geq 0$ and $K_b > 0$ are integer numbers, then the analysis filters can be expressed as

$$H_m(z) = \sum_{q=0}^{Q-1}\left[z^{-Lq}H'_{b1q}(-z^{LQ}) \times \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K_b}q)}z^{-j}E'_j(-z^{2M})\right] \tag{12}$$

where

$$H'_{b1q}(z) = \sum_{k=0}^{K_c-1}(-1)^{K_aq}h_b(kQ+q)z^{-k} \tag{13}$$

for $q = 0, 1, \ldots, (Q-1)$, where $Q = 2K_b$ and $(N_b+1) = QK_c$, and an efficient FRM-CMFB structure results.

Example 1: Design a CMFB with $M = 1024$ channels, using a roll-off factor of $\rho = 0.1$, a maximum bandpass ripple of $A_p = 0.2$ dB, and a minimum stopband attenuation of $A_r = 50$ dB. The standard minimax design would require a prototype filter of order $N = 88865$, which is impractical.

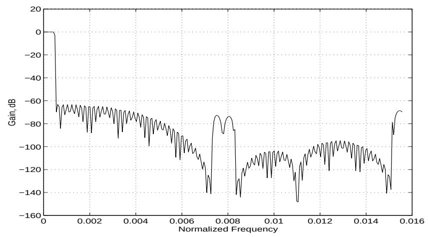
Using the FRM-CMFB structure, with $K_a = 0$ and $K_b = 4$, such that $L = 256$ (which corresponds to $Q = 8$ polyphase components for the base filter) we obtain a prototype filter with a total of $\mathcal{M} = 1147$ coefficients, as described in Table I. The partial magnitude response for the prototype filter can be seen in Figure 3, where one can also see the 16 first channels of the corresponding filter bank.

[2] The general case is addressed, for instance, in [5].

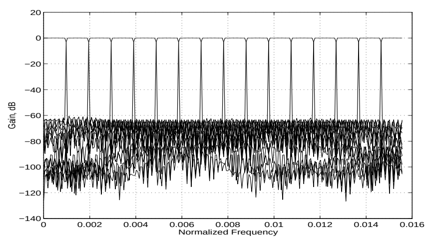
TABLE I

FRM-CMFB CHARACTERISTICS IN EXAMPLE 1.

L	N_b	N_+	N_-	$\mathcal{M}$	A_p	A_r
256	344	801	0	1147	0.08 dB	62 dB



(a)



(b)

Fig. 3. Magnitude responses of Example 1: (a) Prototype filter; (b) Filter bank (bands 0 to 15).

V. THE FRM2-CMFB STRUCTURE

As seen above, the FRM method can be an efficient way to design CMFB prototype filters. There are cases, as the one illustrated in Example 1, that even the FRM-CMFB tends to present reasonably high computational complexity. The structure can then be further simplified if another stage of FRM is used to design the original FRM-CMFB masking filter, yielding the so-called FRM2-CMFB structure. This technique is referred to as a two-stage FRM method, whih is similar to the masking filter factorization introduced in [2].

In the FRM2-CMFB structure, we can then write that

$$H_m(z) = \sum_{n=0}^{N} c_{m,n}((h_{b1}^{I} * h_b^{I'}) * g_1)(n) z^{-n} \tag{14}$$

where $(h_{b1}^{I} * h_b^{I'})(n)$ denotes the convolution of the two interpolated base filters. Notice, however, that this convolution must present an overall interpolation factor that satisfies Eq. (11). For instance, if L is a multiple of L', we can write that

$$H_m(z) = \sum_{i=0}^{N_B} \left[(h_{b1}^{I_2} * h_b')(i) z^{-L'i} \sum_{n=0}^{N_m} c_{m,(n+L'i)} g_1(n) z^{-n} \right] \tag{15}$$

where N_B is the order of the convolution $(h_{b1}^{I} * h_b^{I'})(n)$, and $h_{b1}^{I_2}$ represents the original base filter interpolated by a factor of L/L'. From Eq. (15), the values of $c_{m,(n+L'i)}$ depend only on L', and therefore the two base filters together will not misalign the DCT-IV terms in the masking filter decomposition. We can then rewrite $H_m(z)$ as

$$H_m(z) = \sum_{q=0}^{Q'-1} \Bigg[z^{-L'q} \bar{H}_{b1q}(-z^{L'Q'}) \times \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K_b'}q)} z^{-j} E_j'(-z^{2M}) \Bigg] \tag{16}$$

where Q', L', and K_b' are the respective counterparts of Q, L, and K_b, in the polyphase decomposition of the second base filter, $H_b'(z)$, and $\bar{H}_{b1q}(-z^{L'Q'})$ represents the z transform of the convolutions between $h_{b1}^{I_2}$ and each polyphase component of $H_b'(z)$, the second base filter. It is possible, however, in the z domain to treat each of these convolutions as the cascade of two filters, turning Eq. (16) into

$$H_m(z) = H_{b1}(-z^L) \sum_{q=0}^{Q'-1} \Bigg[z^{-L'q} H_{bq}'(-z^{L'Q'}) \times \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K_b'}q)} z^{-j} E_j'(-z^{2M}) \Bigg] \tag{17}$$

where $H_{bq}'(z)$ is the qth polyphase component of $H_b'(z)$, for $q = 0, 1, \ldots, (Q'-1)$. From this equation, we notice that in the FRM2-CMFB structure, it is then necessary to decompose the second-stage base filter by a factor of $Q' = 2M/L'$, and to introduce a slightly changed version of the interpolated base filter, $H_{b1}(-z^L)$, at the input, as depicted in Figure 4.

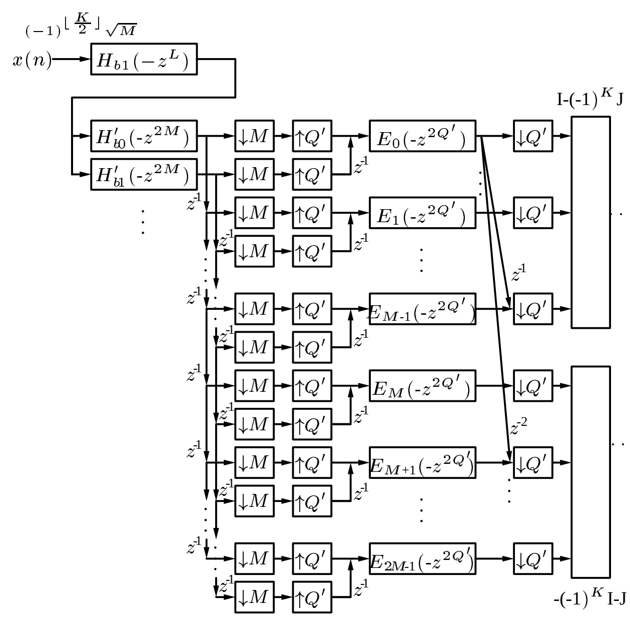
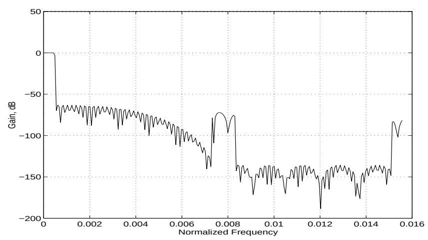


Fig. 4. Block diagram of the FRM2-CMFB (the DCT-IV block is not shown).

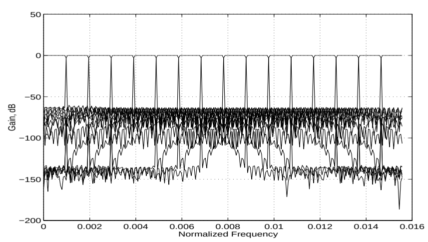
Example 2: In this example, we design the filter bank described in Example 1 using the FRM2-CMFB method. In this case, the original base filter remains unchanged while the masking filter is decomposed into two new filters. For the second-stage base filter the interpolation factor was chosen to be $L' = 16$, corresponding to $Q' = 128$, yielding the filter characteristics shown in Table II and the magnitude responses seen in Figure 5. Thus, in this case, for the complete FRM2-CMFB design only a total of $\mathcal{M} = 459$ coefficients is needed.

TABLE II
FRM2-CMFB CHARACTERISTICS IN EXAMPLE 2.

L'	N'_b	N'_+	N'_-	$\mathcal{M}$	A_p	A_r
16	66	49	0	459	0.02 dB	60 dB



(a)



(b)

Fig. 5. Magnitude responses of Example 2: (a) Prototype filter; (b) Filter bank (bands 0 to 15).

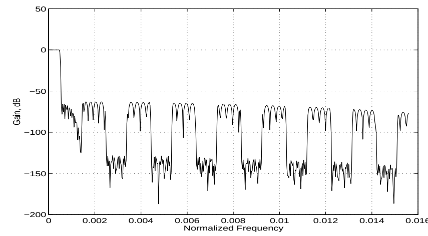
Example 3: In this example, the same filter bank of Example 1 is designed with the FRM2-CMFB method, starting, however, from a FRM-CMFB with $L = M = 1024$, thus requiring only $Q = 2$ polyphase components for the base filter. In such case, the FRM-CMFB is characterized in Table III, where we notice the extremely high order required by the original masking filter. Using the FRM2-CMFB structure to reduce the computational complexity, with $L' = 64$, corresponding to $Q' = 32$, yields the characteristics included in Table IV. We then notice that the total number of coefficients is drastically reduced to $\mathcal{M} = 396$, which is even smaller than the results achieved in Example 2. The magnitude responses for this FRM2-CMFB prototype filter and overall filter bank are seen in Figure 6.

TABLE III
FRM-CMFB CHARACTERISTICS IN EXAMPLE 3.

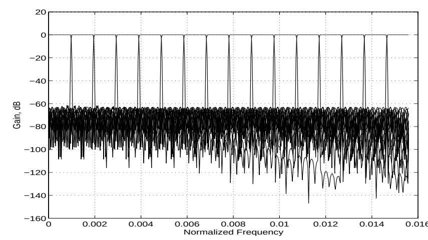
L	N_b	N_+	N_-	$\mathcal{M}$
1024	88	7613	0	7703

TABLE IV
FRM2-CMFB CHARACTERISTICS IN EXAMPLE 3.

L'	N'_b	N'_+	N'_-	$\mathcal{M}$	A_p	A_r
64	116	189	0	396	0.045 dB	60 dB



(a)



(b)

Fig. 6. Magnitude responses of Example 3: (a) Prototype filter; (b) Filter bank (bands 0 to 15).

VI. CONCLUSION

The problem of designing narrowband CMFBs was addressed by introducing the so-called FRM2-CMFB structure. In this structure, a two-stage FRM approach is used to design the CMFB prototype filter, yielding a computationally efficient implementation.

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