A NEW PROCEDURE FOR THE OPTIMIZED DESIGN OF CMFBS BASED ON THE FREQUENCY-RESPONSE MASKING TECHNIQUE

Miguel B. Furtado Jr., Paulo S. R. Diniz, and Sergio L. Netto

Programa de Engenharia Elétrica, COPPE/EPoli/UFRJ
PO BOX 68504, Rio de Janeiro, RJ, 21945-970, BRAZIL
{furtado, diniz, sergioln}@lps.ufrj.br

ABSTRACT

The frequency-response masking (FRM) method allows the design of selective prototype filters for cosine-modulated filter banks (CMFBs) with a reduced number of distinct coefficients. Such methodology may result in filter banks with large number of bands (e.g. 1024 or more) and a simplified optimization procedure, as there are less parameters to adjust. This work introduces a numerically efficient optimization procedure, based on a quasi-Newton algorithm, for designing selective FRM-based CMFBs. The proposed method uses a perfect-reconstruction FRM prototype filter as a starting point and updates the number of bands of the filter bank during the optimization procedure. Examples provided indicate that figures-of-merit, such as intersymbol and intercarrier interferences, for the optimized FRM-CMFB structure are significantly improved without increasing the complexity of the resulting structure.

1. INTRODUCTION

Cosine-modulated filter banks (CMFBs) are widely used in practical multirate systems due to two main aspects [1–6]: First, their complete design is based on a single prototype filter; Second, they have computationally efficient implementations based on fast algorithms for the discrete cosine transform (DCT). For very demanding applications, where maximum selectivity is required, the CMFB prototype filter may present very high order, increasing the computational complexity of the overall structure. A possible design method that may reduce this problem is based on the frequency-response masking (FRM) method [7], which is known to produce sharp linear-phase FIR filters with reduced number of coefficients.

This paper introduces an optimization procedure for the so-called FRM-CMFB prototype filter [8], aiming at the reduction of the stopband total energy or maximum magnitude, with constraints on the intersymbol interference (ISI) and intercarrier interference (ICI) of the overall structure. It is then verified that the reduced number of coefficients required by the FRM approach not only may generate a more efficient structure in terms of number of operations per output sample, but it also leads to a simpler and faster optimization problem. The proposed procedure is based on variations of sequential quadratic programming, using a constrained quasi-Newton method with line search [9]. The new method uses a perfect-reconstruction FRM prototype filter as initial condition and updates the order of the filter to achieve a numerically efficient optimization procedure. The results include lower levels of ISI and ICI for a fixed filter order, or a reduced filter bank complexity for given levels of interferences.

This paper is organized as follows: In Section 2, a description of the cosine-modulated transmultiplexer (TMUX) is given along with the definitions of the associated ISI and ICI figures-of-merit. In Section 3, the FRM-CMFB structure is discussed as an efficient alternative to design highly selective multirate systems. In Section 4, an optimization procedure for the FRM-CMFB structure is introduced, with emphasis given on the numerically efficient features of the proposed algorithm. Finally, Section 5 includes some design examples, illustrating the results achieved with the proposed procedure.

2. THE CMFB AND TMUX SYSTEMS

CMFBs are easy-to-implement structures based on a single prototype filter, whose modulated versions will form the analysis and synthesis subfilters of the complete bank [1]. Usually, the prototype filter for M-band filter bank is specified by its 3 dB attenuation point and the stopband edge at frequencies

$$\omega_{3dB} \approx \frac{\pi}{2M}; \quad \omega_r = \frac{(1+\rho)\pi}{2M} \tag{1}$$

respectively, where ρ controls the amount of overlap between adjacent bands.

Assuming that the prototype filter has an impulse response $h_p(n)$ of order N_p, its transfer function can be expressed as

$$H_p(z) = \sum_{n=0}^{N_p} h_p(n) z^{-n} \tag{2}$$

The impulse response of the analysis and synthesis subfilters are then described by

$$h_m(n) = h_p(n) c_{m,n} \tag{3}$$
$$f_m(n) = h_p(n) \bar{c}_{m,n} \tag{4}$$

for $m = 0, 1, \ldots, (M-1)$ and $n = 0, 1, \ldots, N_p$, where

$$c_{m,n} = 2\cos\left[\frac{(2m+1)(n-N_p/2)\pi}{2M} + (-1)^m \frac{\pi}{4}\right] \tag{5}$$

$$\bar{c}_{m,n} = 2\cos\left[\frac{(2m+1)(n-N_p/2)\pi}{2M} - (-1)^m \frac{\pi}{4}\right] \tag{6}$$

If the prototype filter has $(N_p + 1) = 2KM$ coefficients, then it can be decomposed into $2M$ polyphase components

$$H_p(z) = \sum_{j=0}^{2M-1} z^{-j} E_j(z^{2M}) \tag{7}$$

with $E_j(z)$, for $j = 0, 1, \ldots, (2M-1)$, given by

$$E_j(z) = \sum_{k=0}^{K-1} h_p(2kM + j) z^{-k} \tag{8}$$

Therefore, using the fact that

$$c_{m,(n+2kM)} = (-1)^k c_{m,n} \tag{9}$$

each analysis filter can be written as

$$H_m(z) = \sum_{j=0}^{2M-1} c_{m,j} z^{-j} E_j(-z^{2M}) \tag{10}$$

It can then be shown that all filtering operations can be efficiently implemented through a reduced number of operations per output sample [1]. Clearly, a similar reasoning can also be applied to the synthesis filter bank.

The input-output relationship of the CMFB described above is given by

$$\hat{Y}(z) = \frac{1}{M}\left[T_0(z)Y(z) + \sum_{i=1}^{M-1} T_i(z) Y(ze^{\frac{j2\pi i}{M}})\right] \tag{11}$$

where

$$T_0(z) = \sum_{m=0}^{M-1} F_m(z) H_m(z) \tag{12}$$

$$T_i(z) = \sum_{m=0}^{M-1} F_m(z) H_m(ze^{\frac{-j2\pi i}{M}}) \tag{13}$$

The maximally decimated M-channel TMUX system is a filter bank where the analysis and synthesis blocks are switched, as depicted in Figure 1 [1, 2]. This structure interpolates and filter each input signal, adding the resulting signals on each branch to form a single signal for transmission over a given channel C. At the receiver, the signal is then split back into M-channels to generate the desired M outputs.

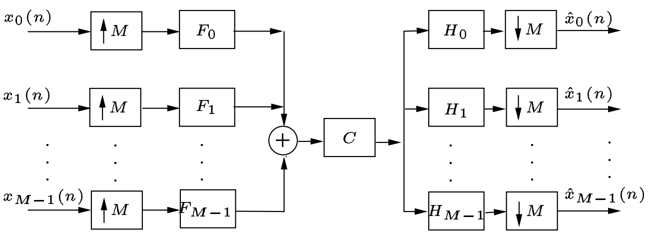

Fig. 1. M-channel maximally decimated TMUX system.

The general relation that describes the transfer functions of the TMUX system is

$$\hat{\mathbf{x}}(z^M) = \frac{1}{M}\mathbf{T}(z^M)\mathbf{x}(z^M) \tag{14}$$

where

$$\hat{\mathbf{x}}(z) = [\hat{X}_0(z)\ \hat{X}_1(z)\ \ldots\ \hat{X}_{M-1}(z)]^T \tag{15}$$

$$\mathbf{x}(z) = [X_0(z)\ X_1(z)\ \ldots\ X_{M-1}(z)]^T \tag{16}$$

$$[\mathbf{T}(z^M)]_{ab} = \sum_{m=0}^{M-1} H_a(ze^{-\frac{j2\pi m}{M}}) F_b(ze^{-\frac{j2\pi m}{M}}) \tag{17}$$

for $a, b = 0, 1, \ldots, (M-1)$. The matrix $\mathbf{T}(z^M)$ is the so-called transfer matrix whose elements, $[\mathbf{T}(z^M)]_{ab}$, represent the transfer function between the interpolated input a and the decimated output b. In a TMUX system, one would be interested in estimating the total ISI and ICI figures of merit which are given by [5]:

$$\text{ISI} = \max_a \left\{ \sum_n [\delta(n) - t_a(n)]^2 \right\} \tag{18}$$

$$\text{ICI} = \max_{a,\omega} \left\{ \sum_{b=0, a\neq b}^{M-1} |[\mathbf{T}(e^{j\omega})]_{ab}|^2 \right\} \tag{19}$$

3. THE FRM-CMFB STRUCTURE

A block diagram of the FRM approach is depicted in Figure 2 [7]. In such scheme, the L-interpolated base filter $H_{b1}(z^L)$ presents a repetitive spectrum which is cascaded by the positive masking filter $G_1(z)$, or order N_m^+, in the upper branch of this realization. Similarly, a complementary version of this repetitive frequency response, $H_{b2}(z^L)$, is processed by the negative masking filter $G_2(z)$, of order, N_m^-, in the lower branch of the realization. In such procedure, both masking filters keep some of the spectrum repetitions within the desired passband, which are then added together to compose the desired frequency response.

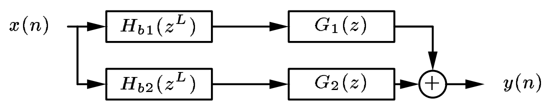

Fig. 2. Block diagram of FRM approach.

To apply the FRM method to design the CMFB prototype filter, let us consider only the upper branch of the FRM structure. Then the transfer functions for the analysis filters become

$$H_m(z) = \sum_{n=0}^{N} c_{m,n} (h_{b1}^I * g_1)(n) z^{-n} \tag{20}$$

where the term $(h_{b1}^I * g_1)(n)$ denotes the convolution between the interpolated base filter and the positive masking filter responses, and N is the overall order of the FRM filter. The key point is to find an efficient structure to calculate the convolution in equation (20), taking into consideration the special property of the cosine functions in equation (9).

In the more general solution, the FRM interpolation factor is written as [8]

$$L = 2K_a M + \frac{M}{K_b} \tag{21}$$

with K_a a nonnegative integer and K_b a positive integer. In such case, $H_m(z)$, after some intricate algebraic manipulations, can be written as [8]

$$H_m(z) = \sum_{q=0}^{Q-1} \left[z^{-Lq} H'_{b1q}(-z^{LQ}) \times \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K_b}q)} z^{-j} E'_j(-z^{2M}) \right] \tag{22}$$

where the modified polyphase components of the interpolated base filter are given by

$$H'_{b1q}(z) = \sum_{k=0}^{K_c-1} (-1)^{K_a q} h_b(kQ+q) z^{-k} \tag{23}$$

for $q = 0, 1, \ldots, (Q-1)$. Based on equation (22), an efficient FRM-CMFB structure results, as detailed in [8].

4. OPTIMIZED FRM-CMFB

The designs of CMFBs aim at the optimization of

$$E_2 = \int_{\omega_r}^{\pi} |H_p(e^{j\omega})|^2 d\omega \tag{24}$$

$$E_\infty = \max_{\omega \in [\omega_r, \pi]} |H_p(e^{j\omega})| \tag{25}$$

which correspond to the total energy and the maximum magnitude value in the filter's stopband, respectively, with ω_r being the stopband edge frequency. In practice, to control the aliasing distortion and the overall direct transfer of the filter bank, the following constraints are introduced

$$1 - \delta_1 \leq |T_0(e^{j\omega})| \leq 1 + \delta_1 \tag{26}$$

$$|T_i(e^{j\omega})| \leq \delta_2 \tag{27}$$

for $i = 1, 2, \ldots, (M-1)$ and $\omega \in [0, \pi]$. In the FRM-CMFB structure, the approximation problem resides on finding a base filter, a positive masking filter (upper branch), and a negative masking filter (lower branch) that optimize E_2 or E_∞ subject to the constraints given by equations (26) and (27).

The functions $T_i(z)$, for $i = 0, 1, \ldots, (M-1)$, required to impose the desired constraints, have an extremely high computational complexity. Some simplifications , however, reduce this problem by rewriting these functions as [10]

$$T_i(z) = Z\left\{\left(e^{\frac{j2\pi in}{M}} h_p(n) * h_p(n)\right)\gamma(n)\right\} \tag{28}$$

with $\gamma(n)$ given by

$$\gamma(n) = \begin{cases} 2M(-1)^c, & \text{for } (N_p - n) = 2Mc,\ c \text{ integer} \\ 0, & \text{otherwise} \end{cases} \tag{29}$$

Due to the symmetry in the modulation function [3], $T_i(z) = T_{M-i}(z)$, and hence, one may evaluate these functions solely for $i = 0, 1, \ldots, \lfloor M/2 \rfloor$, where $\lfloor x \rfloor$ denotes the integer part of x.

4.1. Proposed Algorithm

An efficient procedure for designing optimized CMFB structures was presented in [3]. That procedure, based on an initially optimized CMFB with a small number of bands, M_i, is capable of designing optimized structures with a large number of bands, given by $M_l = 2^c M_i$, where c is a positive integer.

In this work, we extend the above mentioned procedure for the design of FRM-CMFB structures. For simplicity, assuming that the FRM structure is implemented with only the positive branch, and that the non-optimized and the optimized FRM-CMFB prototype filters for a given numbers of bands M are given by] $H_p(z) = H_b(z^L)G_m(z)$ and $\hat{H}_p(z) = \hat{H}_b(z^L)\hat{G}_m(z)$, respectively, the optimization procedure is described by:

Step 1: Make $k = 0$ and $M^{(k)} = M_i$, the initial number of bands of the FRM-CMFB structure. Determine the orders $N_b^{(k)}$ and $N_m^{(k)}$ of the base and positive masking filters at iteration k, respectively, and the interpolation factor L, to obtain a prototype filter of order $N_p^{(k)} = 2KM^{(k)} - 1$ (K is the desired length of the polyphase components of the prototype filter).

Step 2: Initialize all coefficients of $H_b^{(k)}(z)$ as zeros, except the central coefficient, which should be made equal to one, such that $H_b^{(k)}(z) = z^{-N_b^{(k)}/2}$. Also, initialize all coefficients of $G_m^{(k)}(z)$ as zeros, but only the L central coefficients as $\frac{1}{\sqrt{2M^{(k)}}}$.

Step 3: Based on the desired objective function, optimize the FRM-CMFB structure (see next subsection) considering the constraints in equations (26) and (27), obtaining $\hat{H}_b^{(k)}(z^L)$ and $\hat{G}_m^{(k)}(z)$. If $M^{(k)} = M_l$, stop; otherwise, go to next step.

Step 4: Make $k = k+1$ and $M^{(k)} = 2M^{(k-1)}$ and determine the prototype filter (initial solution) for this new filter bank (as the number of bands is doubled) by

$$\begin{aligned} H_p^{(k)}(z) &= \hat{H}_b^{(k-1)}(z^{2L})\hat{G}_m^{(k-1)}(z^2)(1+z^{-1})/2 \\ &= H_b^{(k)}(z^L)G_m^{(k)}(z) \end{aligned} \tag{30}$$

and go back to Step 3.

This procedure is particularly useful when a filter bank with a large number of bands is desired, because it can the designed in steps, gradually increasing the computational complexity. The idea is to use as initial solution for the step k, the optimized solution in step $k-1$, interpolated by a factor of two, with its spectral repetition centered at $\omega = \pi$ partially attenuated by the filter $(1+z^{-1})/2$. In the proposed method, the base and positive masking filters are optimized together, in a single step. A two-step optimization procedure may be implemented, with all FRM sub-filters optimized separately.

4.2. Optimization Algorithms

In this paper, a quasi-Newton algorithm [9] with line search was used as an optimization procedure in Step 3 above. Other option is to use the second algorithm of Dutta and Vidyasagar, as described in [3]. Such algorithm modifies the objective function, to take into account the original objective function and the constraints, simultaneously, each one with its respective weight. Another alternative includes the use of a sequential quadratic programming (SQP) algorithm [9, 11], which solves the general optimization problem by sequentially optimizing a quadratic subproblem. An example of such routine is the `fmincon` command provided in the commercial package MATLAB®.

5. OPTIMIZED DESIGN EXAMPLE

A FRM-CMFB structure with only the positive branch was designed for $M = 32$ bands, $\rho = 1.0$, $\delta_1 \leq 0.0001$ and $\delta_2 \leq 1 \cdot 10^{-05}$. To achieve that number of bands, we started with $M^{(0)} = 2$, and gradually increased it, optimizing the bank for $M^{(k)} = 2^i$, $i = 1, 2, \ldots, 5$, with $L = 2$. We considered both E_2 and E_∞ objective functions. The initial base and positive masking filters orders were chosen $N_b = 8$ and $N_m = 15$, leading to an overall FRM-CMFB prototype filter of order $N_p = 2KM - 1 = 31$ ($K = 8$).

Table 1 summarizes the characteristics of interest for both the standard-CMFB and FRM-CMFB structures, in each step of the procedure. The entry $\mathcal{M}_p$ is the total number of filter coefficients being optimized. Table 2 shows the figures of merit for the optimized standard-CMFB and FRM-CMFB designs ($M = 32$ bands). Clearly, the results are very similar, but, as given in Table 1, the FRM-CMFB required a smaller number of coefficients to implement the bank, yielding a much more efficient optimization procedure. The magnitude responses of both the optimized E_2 and E_∞ FRM prototype filters are shown in Figures 3 and 4, respectively.

Table 1. Characteristics of standard-CMFB and FRM-CMFB structures during the optimization procedure.

structure	N_b	N_{g_1}	N_p	$\mathcal{M}_p$	M
CMFB	-	-	31	16	2
	-	-	63	32	4
	-	-	127	64	8
	-	-	255	128	16
	-	-	511	256	32
FRM-CMFB	8	15	31	16	2
	16	31	63	25	4
	32	63	127	49	8
	64	127	255	97	16
	128	**255**	**511**	**193**	**32**

Table 2. Figures of merit for the optimized standard-CMFB and FRM-CMFB prototype filters.

figures of merit	**Standard**		**FRM**	
	E_∞	E_2	E_∞	E_2
E_2	$1.4\ 10^{-11}$	$5.2\ 10^{-13}$	$1.1\ 10^{-11}$	$4.4\ 10^{-13}$
E_∞ (dB)	-109.8	-100.3	-107.2	-93.2
δ_1	0.0001	0.0001	0.0001	0.0001
δ_2	$8.2\ 10^{-06}$	$2.2\ 10^{-06}$	$5.8\ 10^{-06}$	$8.6\ 10^{-07}$
ICI (dB)	-60.2	-60.2	-60.2	-60.2
ISI (dB)	-62.6	-62.6	-60.2	-60.2

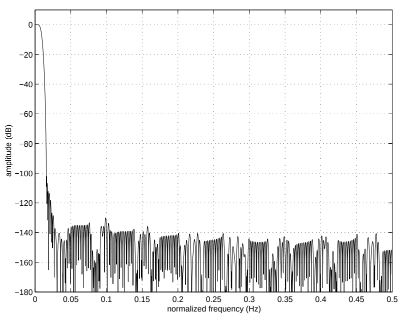

Fig. 3. Magnitude response of the optimized E_2 FRM-CMFB prototype filter for $M = 32$ bands.

6. CONCLUSIONS

A new optimization procedure for CMFB prototype filters based on the frequency-response masking (FRM) approach was proposed.

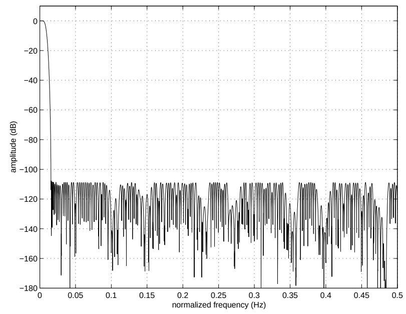

Fig. 4. Magnitude response of the optimized E_∞ FRM-CMFB prototype filter for $M = 32$ bands.

In the numerically efficient algorithm, a perfect-reconstruction prototype filter is used as a starting point to satisfy desired ISI and ICI constraints. The proposed procedure is then based on a quasi-Newton algorithm which optimizes partial filter banks with small number of bands, increasing such number up to the desired value. The result is an efficient procedure that achieves lower interference levels without increasing the complexity of the overall FRM-CMFB structure.

7. REFERENCES

[1] P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Englewood Cliffs, NJ: Prentice Hall, 1993.

[2] N. J. Fliege, *Multirate Digital Signal Processing*. Chichester,NY: John Wiley & Sons, 1994.

[3] T. Saramäki, A generalized class of cosine-modulated filter banks, *Proc. TICSP Workshop on Transforms and Filter Banks*, 336–365, Tampere, Finland, June 1998.

[4] A. Viholainen, T. Saramäki, and M. Renfors, Cosine-modulated filter bank design for VDSL modems, *Proc. IEEE Int. Workshop Intelligent Signal Processing and Communic. Systems*, 143–147, Melbourne, Australia, Nov. 1998.

[5] A. Viholainen, T. Saramäki, and M. Renfors, Nearly-perfect reconstruction cosine-modulated filter bank design for VDSL modems, *Proc. IEEE Int. Conf. Electronics, Circuits, and Systems*, 373–376, Paphos, Greece, Sept. 1999.

[6] J. Alhava and A. Viholainen, Implementation of nearly-perfect reconstruction cosine-modulated filter banks, *Proc. FinSig*, 222–226, Oulu, Finland, 1999.

[7] Y.-C. Lim, Frequency-response masking approach for the synthesis of sharp linear phase digital filters, *IEEE Trans. Circuits and Systems*, CAS-33, 357–364, Apr. 1986.

[8] P. S. R. Diniz, L. C. R. de Barcellos, and S. L. Netto, Design of cosine-modulated filter bank prototype filters using the frequency-response masking approach, *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, VI, P4.6 1–4, Salt Lake City, UT, May 2001.

[9] D. G. Luenberger, *Linear and Nonlinear Programming*, Addison-Wesley Publishing Company, Menlo Park, California, USA, 1989.

[10] M. B. Furtado Jr., P. S. R. Diniz, and S. L. Netto, Optimized cosine-modulated filter banks using the frequency response masking approach, *Proc. IEEE International Telecommunications Symposium*, 143–147, Natal, Brazil, Sept. 2002.

[11] MATLAB Optimization Toolbox: User's Guide, The MathWorks Inc., 1997.