OPTIMIZATION TECHNIQUES FOR COSINE-MODULATED FILTER BANKS BASED ON THE FREQUENCY-RESPONSE MASKING APPROACH

Miguel B. Furtado Jr., Paulo S. R. Diniz, and Sergio L. Netto

Federal University of Rio de Janeiro
Programa de Engenharia Elétrica-COPPE/EP/UFRJ
E-mails: {furtado, diniz, sergioln}@lps.ufrj.br

ABSTRACT

An optimization technique for designing cosine-modulated filter banks using the frequency-response masking approach is proposed. In the given method, we perform minimization in the least-squares and minimax senses, subject to direct and aliasing transfer-function constraints. For optimization, a quasi-Newton algorithm with line search is used, based on sequential quadratic programming. Simplified analytical expressions to impose the interference constraints are proposed. The results are lower levels of aliasing transfers for a pre-determined filter order, or reduced filter complexity for given levels of interference. Specifications that were unfeasible due to lengthy FIR filter requirements are now achieved using the FRM-CMFB design.

1. INTRODUCTION

Cosine-modulated filter banks (CMFBs) are commonly used in practice due to their simple design, based on a single prototype filter, and efficient implementation [1]. For very demanding applications where maximum selectivity is required, the CMFB prototype filter tends to present very high order, thus increasing the computational complexity of the overall structure. One can then use the frequency-response masking (FRM) [2] approach to design the CMFB prototype filter. This technique is known to produce sharp linear-phase FIR filters with reduced number of coefficients, resulting in the so-called FRM-CMFB structure [3]. This paper presents an optimization procedure of the FRM-CMFB prototype filter aiming at the reduction of the maximum stopband attenuation or the stopband energy, with constraints on the direct, $T_0(z)$, and aliasing, $T_i(z)$, CMFB transfer functions. It is then verified that the reduced number of FRM coefficients also leads to a simpler and faster optimization problem. The optimization procedure is based on variations of sequential quadratic programming (SQP), using a constrained quasi-Newton method with line search. A simplified analytical derivation of the interference constraints is given, which greatly speeds up the optimization procedure. The results include lower levels of distortion for the overall transfer for a fixed filter order, or reduced filter bank complexity for given level of interferences.

The remaining of the paper is organized as follows: In Sections 2 and 3, descriptions of the CMFB structure and FRM approach are given. In Section 4, the FRM-CMFB implementation is then presented as an alternative to design highly selective filter banks. In Section 5, the FRM-CMFB optimization procedure is presented, with emphasis given on a simplified analytical derivation for the interference constraints in Section 6. Section 7 includes some design examples, showing improved results achieved with the optimized FRM-CMFB structure.

2. CMFB SYSTEM

CMFBs are easy-to-implement structures based on a single prototype filter, whose modulated versions will form the analysis and synthesis subfilters of the complete bank [1]. The CMFB prototype filter for M-band filter bank is specified by its 3 dB attenuation point and the stopband edge at frequencies

$$\omega_{3dB} \approx \frac{\pi}{2M}; \quad \omega_s = \frac{(1+\rho)\pi}{2M} \tag{1}$$

respectively, where ρ is the so-called roll-off factor.

Assuming that the prototype filter has an impulse response $h_p(n)$ of order N_p, its transfer function is expressed as

$$H_p(z) = \sum_{n=0}^{N_p} h_p(n) z^{-n} \tag{2}$$

The impulse response of the analysis and synthesis filters are

$$h_m(n) = h_p(n) c_{m,n} \tag{3}$$

$$f_m(n) = h_p(n) \bar{c}_{m,n} \tag{4}$$

for $m = 0, 1, \ldots, (M-1)$ and $n = 0, 1, \ldots, N_p$, where

$$c_{m,n} = 2\cos\left[\frac{(2m+1)(n-N_p/2)\pi}{2M} + (-1)^m \frac{\pi}{4}\right] \tag{5}$$

$$\bar{c}_{m,n} = 2\cos\left[\frac{(2m+1)(n-N_p/2)\pi}{2M} - (-1)^m \frac{\pi}{4}\right] \tag{6}$$

If the prototype filter has $(N_p+1) = 2KM$ coefficients, then it can be decomposed into $2M$ polyphase components

$$H_m(z) = \sum_{j=0}^{2M-1} c_{m,j} z^{-j} E_j(-z^{2M}) \tag{7}$$

with $E_j(z)$, for $j = 0, 1, \ldots, (2M-1)$, given by

$$E_j(z) = \sum_{k=0}^{K-1} h_p(2kM+j) z^{-k} \tag{8}$$

what yields a computationally efficient CMFB realization [1].

The CMFB above has an input-output relation described by

$$\hat{Y}(z) = \frac{1}{M}\left[T_0(z)Y(z) + \sum_{i=1}^{M-1} T_i(z)Y(ze^{\frac{j2\pi i}{M}})\right] \quad (9)$$

where $T_0(z)$ is the direct transfer function, which must be the unique term in an alias-free design, and all other $T_i(z)$, represent the aliasing transfer functions, which are expressed by

$$T_0(z) = \sum_{m=0}^{M-1} F_m(z)H_m(z) \quad (10)$$

$$T_i(z) = \sum_{m=0}^{M-1} F_m(z)H_m(ze^{\frac{-j2\pi i}{M}}) \quad (11)$$

3. FRM APPROACH

The FRM approach uses a complementary pair of interpolated linear-phase FIR filters. The base filter, $H_b(z)$, with group-delay $\frac{N}{2}$, and its complementary version, $H_{b_c}(z)$, are interpolated by a factor L, to form sharp transition bands, at the cost of introducing multiple passbands on each response. These repetitive bands are then filtered out by the so-called positive and negative masking filters, $G_1(z)$ and $G_2(z)$ respectively, and added together to compose the desired filter, given by [2]

$$H_f(z) = H_b(z^L)G_1(z) + (z^{-NL/2} - H_b(z^L))G_2(z) \quad (12)$$

4. FRM-CMFB STRUCTURE

An efficient FRM-CMFB joint structure can be derived if the FRM interpolator factor can be expressed as $L = 2K_aM + \frac{M}{K_b}$, with $K_a \geq 0$ and $K_b > 0$ being integer numbers [3]. In such case, using solely the upper branch on the FRM scheme, the mth analysis filter can be written as

$$H_m(z) = \sum_{q=0}^{Q-1}\left[z^{-Lq}H'_{b1q}(-z^{LQ}) \times \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K_b}q)} z^{-j}E'_j(-z^{2M})\right] \quad (13)$$

where

$$H'_{b1q}(z) = \sum_{k=0}^{K_c-1} (-1)^{K_a q} h_b(kQ+q)z^{-k} \quad (14)$$

for $q = 0, 1, \ldots, (Q-1)$, where $Q = 2K_b$ is the number of polyphase components for the FRM base filter, and also

$$E'_j(z) = \sum_{k=0}^{K_d-1} g_1(2kM+j)z^{-k} \quad (15)$$

for $j = 0, 1, \ldots, (2M-1)$, with $i = kQ + q$ and $(N_b + 1) = QK_c$, where N_{g_1} is the order and $g_1(n)$ are the coefficients of the masking filter. Eq. (13) leads to the so-called FRM-CMFB efficient structure described in [3].

5. OPTIMIZATION

Standard optimization goals for the CMFB prototype filter are to minimize the objective functions

$$E_2 = \int_{\omega_r}^{\pi} |H_p(e^{j\omega})|^2 d\omega; \quad E_\infty = \max_{\omega \in [\omega_r, \pi]} |H_p(e^{j\omega})| \quad (16)$$

which correspond to the total energy and the maximum magnitude value in the filter's stopband, respectively. In practice, to control the aliasing distortion and the overall direct transfer of the filter bank, the following constraints are introduced

$$1 - \delta_1 \leq |T_0(e^{j\omega})| \leq 1 + \delta_1 \quad (17)$$

$$|T_i(e^{j\omega})| \leq \delta_2 \quad (18)$$

for $i = 1, 2, \ldots, (M-1)$ and $\omega = \in [0, \pi]$.

In the FRM-CMFB structure, the prototype filter $H_p(z)$ is as given in eq. (12), and the approximation problem resides on finding a base filter, a positive masking filter (upper branch), and a negative masking filter (lower branch) that optimize E_2 or E_∞ subject to the constraints given by eqs. (17) and (18). In this work, for the optimization we used the MATLAB® [5] command `fmincon`. The gradient vector was determined analytically to reduce computational burden during optimization procedure. The evaluation of the constraints on $T_0(z)$ and $T_i(z)$, given in eqs. (17) and (18), can be significantly simplified as described below.

6. ANALYTICAL FORMULATION OF CONSTRAINTS

In the z domain, eqs. (3) and (4) become [6]

$$H_m(z) = \alpha_m\beta_m H_p(ze^{\frac{-j(2m+1)\pi}{2M}}) + \alpha_m^*\beta_m^* H_p(ze^{\frac{j(2m+1)\pi}{2M}}) \quad (19)$$

$$F_m(z) = \alpha_m^*\beta_m H_p(ze^{\frac{-j(2m+1)\pi}{2M}}) + \alpha_m\beta_m^* H_p(ze^{\frac{j(2m+1)\pi}{2M}}) \quad (20)$$

for $m = 0, 1, \ldots, (M-1)$, with

$$\alpha_m = e^{\frac{j(-1)^m\pi}{4}}; \quad \beta_m = e^{\frac{-jN_p(2m+1)\pi}{4M}} \quad (21)$$

Using these relations, the functions $T_i(z)$ can be written as

$$\begin{aligned} T_i(z) = \sum_{m=0}^{M-1}\Big[& {\beta_m}^2 H_p(ze^{\frac{-j(2m+1)\pi}{2M}})H_p(ze^{-\frac{j2\pi i}{M}}e^{\frac{-j(2m+1)\pi}{2M}}) + \\ & {\beta_m^*}^2 H_p(ze^{\frac{j(2m+1)\pi}{2M}})H_p(ze^{-\frac{j2\pi i}{M}}e^{\frac{j(2m+1)\pi}{2M}}) + \\ & {\alpha_m^*}^2 H_p(ze^{\frac{-j(2m+1)\pi}{2M}})H_p(ze^{-\frac{j2\pi i}{M}}e^{\frac{j(2m+1)\pi}{2M}}) + \\ & {\alpha_m}^2 H_p(ze^{\frac{j(2m+1)\pi}{2M}})H_p(ze^{-\frac{j2\pi i}{M}}e^{-\frac{j(2m+1)\pi}{2M}})\Big] \end{aligned}$$

since $({\alpha_m}^2 + {\alpha_m^*}^2) = 0$ and $\alpha_m \alpha_m^* = 1$, $\forall m$. Using eq. (2) and the definitions $W_i = e^{\frac{j2\pi i}{M}}$ and $W_c = e^{\frac{j(2m+1)\pi}{2M}}$, we get

$$T_i(z) = \sum_{m=0}^{M-1}\Bigg[{\beta_m}^2\left(\sum_{n=0}^{N_p} h_p(n)z^{-n}W_c^n\right)\left(\sum_{l=0}^{N_p} h_p(l)z^{-l}W_i^lW_c^l\right) + {\beta_m^*}^2\left(\sum_{n=0}^{N_p} h_p(n)z^{-n}W_c^{-n}\right)\left(\sum_{l=0}^{N_p} h_p(l)z^{-l}W_i^lW_c^{-l}\right) + {\alpha_m^*}^2\left(\sum_{n=0}^{N_p} h_p(n)z^{-n}W_c^{n}\right)\left(\sum_{l=0}^{N_p} h_p(l)z^{-l}W_i^lW_c^{-l}\right) + {\alpha_m}^2\left(\sum_{n=0}^{N_p} h_p(n)z^{-n}W_c^{-n}\right)\left(\sum_{l=0}^{N_p} h_p(l)z^{-l}W_i^lW_c^{l}\right)\Bigg]$$

$$= \sum_{n=0}^{2N_p}\left[a_i(n)z^{-n}\sum_{m=0}^{M-1}\left({\beta_m}^2W_c^n + {\beta_m^*}^2W_c^{-n}\right)\right] + \sum_{n=0}^{N_p}\sum_{l=0}^{N_p}\left[h_p(n)h_p(l)z^{-(n+l)}W_i^l \times \sum_{m=0}^{M-1}\left({\alpha_m^*}^2W_c^{(n-l)} + {\alpha_m}^2W_c^{-(n-l)}\right)\right]$$

where $a_i(n)$ is defined as

$$Z\left\{(W_i^n h_p(n)) * h_p(n)\right\} = \sum_{n=0}^{2N_p} a_i(n)z^{-n} \tag{22}$$

Now, using that (see Appendix)

$$\gamma(n) = \sum_{m=0}^{M-1}\left({\beta_m}^2W_c^n + {\beta_m^*}^2W_c^{-n}\right) = \begin{cases} 0, & (N_p - n) \neq 2Mc \\ 2M(-1)^c, & (N_p - n) = 2Mc,\ c \in \mathbb{Z}\end{cases} \tag{23}$$

$$\Gamma(n-l) = \sum_{m=0}^{M-1}\left({\alpha_m^*}^2W_c^{(n-l)} + {\alpha_m}^2W_c^{-(n-l)}\right) = \begin{cases} 0, & (n-l) \neq (2c+1)M \\ 2M(-1)^c, & (n-l) = (2c+1)M,\ c\in\mathbb{Z}\end{cases} \tag{24}$$

with $\Gamma(n-l) = -\Gamma(l-n)$, $T_i(z)$ can be simplified to

$$T_i(z) = \sum_{n=0}^{2N_p} a_i(n)z^{-n}\gamma(n) + \sum_{n=0}^{N_p}\sum_{l=0}^{N_p} h_p(n)h_p(l)z^{-(n+l)}W_i^l\Gamma(n-l) = \sum_{n=0}^{2N_p} a_i(n)z^{-n}\gamma(n) \tag{25}$$

since

$$\sum_{n=0}^{N_p}\sum_{l=n}^{N_p} h_p(n)h_p(l)z^{-(n+l)}W_i^l\Gamma(n-l) = -\sum_{n=0}^{N_p}\sum_{l=n}^{N_p} h_p(n)h_p(l)z^{-(n+l)}W_i^n\Gamma(n-l) \tag{26}$$

In this way, all functions $T_i(z)$ can be evaluated using this simplification, convolving the prototype filter with its complex modulated version, as follows:

$$T_i(z) = Z\left\{(W_i^n h_p(n) * h_p(n))\,\gamma(n)\right\} \tag{27}$$

for $i = 0, \ldots, (M-1)$. Due to the symmetry $T_i(z) = T_{M-i}(z)$, the $T_i(z)$ can be determined only for $i = 0, \ldots, \lfloor M/2 \rfloor$, where $\lfloor x \rfloor$ denotes the integer part of x. If the prototype filter is linear-phase, then, the functions $T_i(z)$ can be written as

$$T_i(z) = z^{-N_p}2M\left[a_i(N_p) + \sum_{l=1}^{K-1} a_i(N_p - 2Ml)(-1)^l\left(z^{2Ml} + z^{-2Ml}\right)\right] \tag{28}$$

Table 1 shows the number of floating-point multiplications associated to eqs. (11) and (28), assuming that the prototype of order $N_p = 2KM - 1$ has linear phase. The entries CM and PM stand for cosine modulation and polynomial multiplication. The last line in this table considers the total real and imaginary calculations. Clearly, if $K >> 1$ and $M >> 1$, the total number of multiplications become $4K^2M^3$ and $2K^2M$, respectively, leading to a reduction factor of $r = \frac{1}{2M^2}$ with the simplified computations.

Table 1: Computational complexity for the standard and simplified formulations.

step	**standard**	**simplified**
CM	$4KM$	-
PM	$KM(2KM+1) \times M$	$KM(K+1)$
Total	$2KM^2(2KM + 1 + \frac{3}{M})$	$2KM(K+1)$

For the FRM-CMFBs, $T_i(z)$ must consider the entire FRM structure given in eq. (12). For simplicity, assuming that only the upper branch is used, the prototype filter becomes $H_p(z) = H_b(z^L)G_1(z)$ and the coefficients $a_i(n)$ in eq. (28) are given by

$$a_i(N_p - 2Mk) = \sum_{\tau=0}^{N_p-2Mk} e^{\frac{j2\pi i\tau}{M}} h_p(N_p - 2Mk - \tau)h_p(\tau) = \sum_{\tau=0}^{N_p-2Mk} e^{\frac{j2\pi i\tau}{M}} h_p(2Mk + \tau)h_p(\tau) \tag{29}$$

7. DESIGN EXAMPLE

A SQP algorithm was applied on both direct-form and FRM realizations of a CMFB, to optimize their performances with respect to E_2 and to observe if the results can get closer.

The example compares the realization of a CMFB with $M = 64$ bands and $\rho = 1$, with overall order of the prototype filters set to $N_p = 2KM - 1 = 1023$. For the direct-form, a factor of $K = 6$ was used. For the FRM structure, a factor $L = 16$ was employed, allowing one to discard the lower branch of the FRM diagram and reduce considerably the number of coefficients used. The orders of the base and positive masking filters were $N_b = 55$ and $N_{g_1} = 143$, respectively.

The parameters of interest for both structures, namely passband ripple, δ_1, aliasing interference, δ_2, minimum stopband attenuation, E_∞, and stopband energy, E_2, are summarized in Table 2. The magnitude responses of both the optimized FRM prototype filter and the complete FRM-CMFB are presented in Figures 1 and 2, respectively.

Table 2: Figures of merit for the optimized prototype filters.

Figures of Merit	**Direct Form**	**FRM**
# coefficients	512	100
δ_1	0.0001	0.0001
δ_2 (dB)	-123.5	-112.9
E_∞ (dB)	-102.3	-98.2
E_2	$1.3 \cdot 10^{-13}$	$4.0 \cdot 10^{-13}$

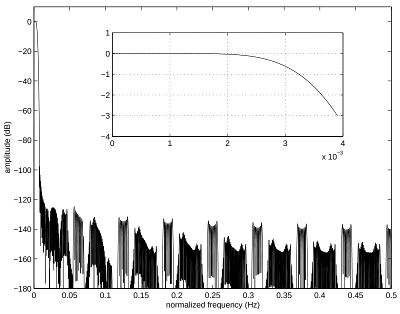

Figure 1: Magnitude response of optimized FRM prototype filter.

8. CONCLUSIONS

A new design procedure for optimizing the FRM-CMFB prototype filter was presented. In the proposed method, a quasi-Newton method is used to perform minimization of the maximum value of the magnitude response or the stopband energy transfer within the filter's stopband. Constraints related to direct transfer and aliasing distortions are considered, in an extremely simplified manner. The result is a procedure that yields very efficient filter banks with respect to several figures of merit, including the number of coefficients capitalized by the FRM-CMFB structure.

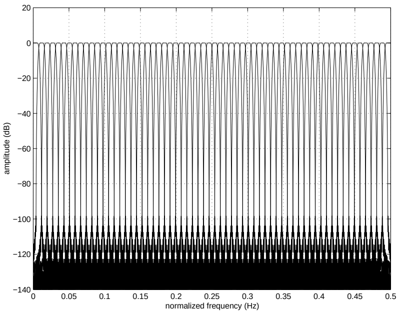

Figure 2: Magnitude response of optimized FRM-CMFB.

APPENDIX

A detailed derivation of eq. (23) appears in [4]. Similarly, using the definition of α_m in eq. (21), the function $\Gamma(n-l)$, defined in eq. (24), can be written as

$$
\begin{aligned}
\Gamma(n-l) &= -je^{\frac{j\pi(n-l)}{2M}} \sum_{m=0}^{M-1} (-1)^m e^{\frac{jm\pi(n-l)}{M}} + \\
&\quad\; je^{-\frac{j\pi(n-l)}{2M}} \sum_{m=0}^{M-1} (-1)^m e^{-\frac{jm\pi(n-l)}{M}} \\
&= -je^{\frac{j\pi(n-l)}{2M}} \left[\frac{1-\left(-e^{\frac{j\pi(n-l)}{M}}\right)^M}{1-\left(-e^{\frac{j\pi(n-l)}{M}}\right)} \right] + \\
&\quad\; je^{-\frac{j\pi(n-l)}{2M}} \left[\frac{1-\left(-e^{-\frac{j\pi(n-l)}{M}}\right)^M}{1-\left(-e^{-\frac{j\pi(n-l)}{M}}\right)} \right] \\
&= -\frac{\sin \pi(n-l)}{cos\frac{\pi(n-l)}{2M}}
\end{aligned}
$$

resulting in eq. (24).

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