TIME-DOMAIN CONSTRAINTS FOR THE DESIGN OF FRM-BASED COSINE-MODULATED AND MODIFIED DFT FILTER BANKS WITH LARGE NUMBER OF BANDS AND ZERO INTERSYMBOL INTERFERENCE

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) is a transmultiplex (TMUX) technique characterized by a low computational complexity and reduced inter-symbol/inter-carrier interferences (ISI/ICI). Recently, new TMUX structures based on multicarrier modulation have been proposed aiming at even better performances. Such new structures include a cosine-modulated filter bank (CMFB) and a modified DFT filter bank (MDFTB). This work introduces design techniques for the CMFB and MDFTB structures with a large number of subbands (several hundreds or more), zero ISI, and small levels of ICI for most practical applications. The design is based on the concept of an Mth-band Nyquist filter, which inherently yields zero ISI and a highly selective filter bank, and in the FRM structure enabling one to synthesize FIR filters with a significantly reduced number of coefficients. The prototype filter is optimized to minimize the resulting ICI. A design example is included to illustrate the benefits of the proposed technique in filter bank applications demanding a very large number of channels.

1. INTRODUCTION

During the past few years, the area of digital communications has experienced a growing interest on the research of filter banks, due to their inherent subband processing capability. Such structures, when applied to multiple input/output digital communications systems, are known as transmultiplexers (TMUXes) [1]. The TMUX may turn the task of non-flat channel equalization simpler by reducing the computational complexity, even in hostile environments such as the mobile communications channels. Currently, the most widely employed TMUX technique has been the orthogonal frequency division multiplexing (OFDM) [2], which has a low computational complexity and yields an easy equalization process when using a proper cyclic prefix. Such a prefix, however, greatly reduces the OFDM payload. In order to reduce the prefix, synthesis schemes based on the use of multirate filter banks with memory were introduced, imposing some correlation between the transmitted data symbols in each TMUX subchannel. Such alternative TMUX schemes are based, for instance, on the cosine modulated filter banks (CMFBs) [3, 4] or the modified DFT banks (MDFTBs) [5].

Modern communications systems employing OFDM may require a large number of subbands in order to model each subchannel with a flat response and to allow a simple equalization procedure. Such a demand for a large number of subbands turns the design of CMFB or MDFTB prototype filters into a very complicated procedure. This paper proposes a new way to accomplish this task in an efficient manner, with reduced memory and computational requirements, just by imposing time-domain constraints in the optimization of the prototype filter. The main idea is to choose a prototype filter structure based on the frequency response masking (FRM) approach [6] as a building block to form an Mth-band Nyquist filter when cascaded with itself. The resulting prototype filter provides a TMUX structure with zero inter-symbol interference (ISI) and controlled levels of inter-carrier interference (ICI), when transmitting over a perfect channel. For non-perfect channels, as in non-flat and/or time-varying channels, an equalizer is required in order to reduce the resulting levels of ISI and ICI. A similar design approach, though operating in the frequency-domain, was proposed in [7] which considers the simultaneous minimization of the ISI and ICI. In the present paper, however, a null ISI level is inherent to the structure, whereas the ICI level is directly controlled by the stopband attenuation of the prototype filter.

This paper is organized in the following manner: Section 2 reviews the general idea of Nyquist filters and the FRM approach, establishing conditions to design an $2M$th-band Nyquist filter as a cascade of two identical FRM subfilters. Section 3 presents a brief discussion about the filter banks based on the CMFB and MDFTB approaches. We also discuss the optimization details for attaining improved prototype filters for these structures in the least-squares (LS) sense since such an objective function leads to systems with reduced ICI for a fixed ISI and a given prototype-filter order. Section 4 presents a numerical example where a filter bank with a large number of bands and reduced roll-off factor is desired. The prototype filter is designed using both the direct-form approach and the proposed FRM design with time-domain constraints.

2. FRM-BASED NYQUIST FILTERS

M-band Nyquist filters are a class of linear-phase finite impulse response (FIR) filters whose impulse response can be described by [8, 9]

$$h_n(N) = 1/M$$
$$h_n(N+lM) = 0, \quad l = \pm 1, \pm 2, \ \ldots \qquad (1)$$

for a given filter order $2N$ and an interpolation factor M. For the later use, the corresponding transfer function of order N is denoted by $H_n(z)$. Such a class of filters is suitable for digital communications when the data symbols are upsampled by M and filtered by the Nyquist filter. In such cases, there will be no ISI in the received sequence provided that both the transmitter and receiver are synchronized and the channel is of the form $C(z) \equiv z^{-d}$, where d is the overall delay. Hence, the receiver needs only to downsample the received information and recover the original sequence in a very straightforward manner. If the original sampling period of the upsampled signal is T_s, then the data symbols will be spaced in time by $T = MT_s$ and there will be many choices for the design of the Nyquist filter if the channel has bandwidth $W > 1/2T$.

Depending on the application at hand, TMUXes with large number of subchannels are necessary, thereby demanding the optimization of very long CMFB or MDFTB prototype filters. Such optimization problems may not even be feasible due to the computational complexity involved. Another issue is the need for a high computational storage capacity, since efficient optimization algorithms, like the sequential quadratic programming (SQP) [10] used in this paper, require the computation of a Hessian matrix (its inverse or an approximation of it), whose size is about the square of the number of parameters being optimized. To overcome these problems, filter banks designed with the FRM [6] approach were proposed in [7,11], resulting in a remarkable reduction in the number of parameters to be optimized. In these works, the price paid for using an FRM prototype filter with an LS (or minimax) characteristic is a slightly higher stopband energy (or stopband ripple) when compared to the direct-form realization for the same overall filter order.

The FRM overall transfer function is given by [6]

$$H_p(z) = H_b(z^L)G_1(z) + \left(z^{-N_bL/2} - H_b(z^L)\right)G_2(z), \quad (2)$$

where $H_b(z)$ is the so-called base filter, $G_1(z)$ and $G_2(z)$ are the masking filters of the upper and lower FRM branches, respectively, and L is the interpolation factor. This structure is suitable for designing a filter with sharp transition band and large passband. For a narrowband prototype filter, which is required by a filter bank with a large number of subbands, the lower branch is not necessary, and the overall filter can be written as

$$H_p(z) = H_b(z^L)G_1(z), \quad (3)$$

which resembles the classic interpolator structure [12].

In [7], it has been verified analytically that the ISI of an M-channel CMFB is directly related to ${H_p}^2(z)$. Hence, to achieve a null ISI,

$$H_n(z) = {H_p}^2(z) \quad (4)$$

must be a Nyquist filter. Then, introducing the time-domain constraints on the prototype filter design to force ${H_p}^2(z)$, with $H_p(z)$ as described by equation (3) and with order $N_p = (2KM-1)$, to satisfy equation (4), such constraints become

$$h_n(N_p - 2Ml) = \begin{cases} 1/(2M), & \text{if} \quad l = 0 \\ 0, & \text{if} \quad l = \pm 1, \ldots, \pm(K-1), \end{cases} \quad (5)$$

where $1/(2M)$ is a simple scaling factor, and $h_p(n)$ and $h_n(m)$ are the impulse responses of the prototype and the Nyquist filters, satisfying equations (3) and (4), respectively, such that

$$h_p(n) = \sum_{\nu=0}^{\lfloor n/L \rfloor} h_b(\nu) g_1(n - L\nu) \quad (6)$$
$$h_n(m) = \sum_{\tau=0}^{m} h_p(m-\tau) h_p(\tau), \quad (7)$$

where $0 \leq m \leq N_p$ and $h_p(n) = 0$ for $n < 0$ and $n > N_p$. It is worth mentioning that in this case $H_n(z)$ is a $2M$th-band Nyquist filter. The idea of imposing time-domain constraints for ISI free filters was treated in [9] and [13], where the last focused on the direct-form prototype filter for CMFBs and the former developed a cascaded structure to design a general Mth-band filter. The building block for the Nyquist filter treated here is represented in Figure 1.

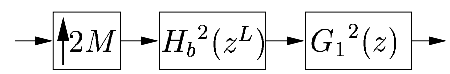

Figure 1: $2M$th-band Nyquist filter composed by FRM subfilters.

Given the nonlinear equality constraints in equation (5)[1], one can determine the first-order derivatives, required by an SQP-like optimization algorithm, as

$$\frac{\partial h_n(N_p - 2Ml)}{\partial h_b(n')} = \sum_{n=0}^{N_p} \frac{\partial h_n(N_p - 2Ml)}{\partial h_p(n)} \frac{\partial h_p(n)}{\partial h_b(n')}, \quad (8)$$
$$\frac{\partial h_p(N_p - 2Ml)}{\partial g_1(n')} = \sum_{n=0}^{N_p} \frac{\partial h_n(N_p - 2Ml)}{\partial h_p(n)} \frac{\partial h_p(n)}{\partial g_1(n')}, \quad (9)$$

for $l = 0, \ldots, K-1$, with

$$\frac{\partial h_n(N_p - 2Ml)}{\partial h_p(n)} = 2\left(h_p(n + 2Ml) + h_p(n - 2Ml)\right), \quad (10)$$

$$\frac{\partial h_p(n)}{\partial h_b(n')} = c_1 \left[g_1\left(n - Ln'\right) + g_1\left(n - L(N_b - n')\right)\right], \quad (11)$$
$$\frac{\partial h_p(n)}{\partial g_1(n')} = c_2 \left[h_b\left(\frac{n-n'}{L}\right) + h_b\left(\frac{n+n'-N_{g_1}}{L}\right)\right], \quad (12)$$

where

$$c_1 = \begin{cases} 1/2, & \text{if } n' = N_b/2 \ (N_b \text{ even}) \\ 1, & \text{otherwise} \end{cases}$$
$$c_2 = \begin{cases} 1/2, & \text{if } n' = N_{g_1}/2 \ (N_{g_1} \text{ even}) \\ 1, & \text{otherwise} \end{cases} \quad (13)$$

for $n = 0, 1, \ldots, (N_p - 1)/2$. Here, we are assuming that the prototype is a symmetric filter, that is, $h_p(n) = h_p(N_p - n)$. Also, $h_b(n') = 0$, $\forall n' \notin \mathbb{Z}$, the set of integer numbers, and $h_b(n') = 0$, $\forall (n' < 0 \ \cup \ n' > N_b)$, and $g_1(n') = 0$, $\forall (n' < 0 \ \cup \ n' > N_{g_1})$.

[1] The coeffi cients of $H_n(z)$ are quadratic with respect to the FRM subfi lter coeffi cients.

A direct benefit from using such time-domain constraints is that the overall computational complexity to determine them depends only on K and not on M. The same does not apply for problems considering frequency-domain constraints, like in [7]. Therefore, when optimizing filter banks with a large number of bands, one should consider using those time-domain restrictions.

3. CMFB AND MDFTB OPTIMIZED STRUCTURES

The CMFB and MDFTB structures are entirely defined by a single prototype filter $H_p(z)$ of order $N_p = (2KM-1)$, where M is the number of subchannels or subbands. It can be shown that the CMFB and MDFTB TMUXes can use the same prototype filter obtaining similar performances. The Mth-band analysis and synthesis CMFB filters are cosine-modulated versions of the prototype filter, which can be described by

$$H_m(z) = \alpha_m \beta_m^{N_p/2} H_p(z\beta_m) + \alpha_m^* \beta_m^{-N_p/2} H_p(z\beta_m^*) \quad (14)$$

$$F_m(z) = \alpha_m^* \beta_m^{N_p/2} H_p(z\beta_m) + \alpha_m \beta_m^{-N_p/2} H_p(z\beta_m^*) \quad (15)$$

where $\alpha_m = e^{j(-1)^m \frac{\pi}{4}}$ and $\beta_m = e^{-j\frac{(2m+1)\pi}{2M}}$ for $m = 0, 1, \ldots, M-1$, and * denotes the complex conjugate operator.

On the other hand, the Mth-band MDFTB analysis and synthesis filters present a shift on each subcarrier frequency equal to half the subchannel bandwidth, and a small phase rotation when compared to an $(M/2)$th-band CMFB, that is,

$$H_m(z) = \frac{1}{2}\gamma_m^{N_p} H_p(z\gamma_m{}^2) \quad (16)$$

$$F_m(z) = \frac{1}{2}\gamma_m^{-N_p} H_p(z\gamma_m^{*\,2}) \quad (17)$$

with $\gamma_m = e^{j\frac{m\pi}{M}}$.

It was empirically verified in [14] that the LS objective function leads to the lowest ICI in a CMFB-based TMUX for a fixed prototype-filter order and ISI level if the optimization process does not take into account the ICI interference (which is the case). Therefore, we optimize the prototype filter subject to the stopband energy, that can be implemented with a discrete objective function given by

$$E_2 = \sum_{\omega_i \in [\omega_s, \pi]} |H_p(e^{j\omega_i})|^2 \Delta_\omega, \quad (18)$$

where ω_s is the stopband frequency edge, $H_p(e^{j\omega_i})$ is the frequency response of the prototype filter at the discrete frequency point ω_i, and Δ_ω is the given frequency grid interval.

4. NUMERICAL EXAMPLE OF OPTIMIZATION

To illustrate the robustness of the proposed procedure for designing ISI-free CMFB and MDFTB structures, specially when a large number of subbands is desired, an M-channel filter bank design example is provided. For an ideal bandwidth given by $\omega_{3dB} = \pi/(2M)$, the passband and stopband cutoff frequencies become $\omega_p = (1-\rho)\pi/(2M)$ and $\omega_r = (1+\rho)\pi/(2M)$, respectively, where ρ is the so-called roll-off factor. The prototype filter specifications are given by:

$$\begin{cases} \text{ISI} &= 0 \\ \text{ICI} &= \text{minimum attainable} \\ M &= 512 \\ K &= 20 \\ \rho &= 0.5. \end{cases} \quad (19)$$

In such a case, the prototype filter order is $N_p = (2KM-1) = 20479$. For a direct-form prototype filter, such an order would correspond to a total of $\mathcal{N} = 10240$ filter parameters to be optimized. There are many choices for the settings of the FRM subfilters, but one that requires only a total of $\mathcal{N} = 271$ parameters is described in Table 1. In this table, N_b is the base-filter order, N_1 is the masking filter order, N_p is the overall filter order, and L corresponds to the FRM interpolation factor. In addition, $\mathcal{N}$ indicates the number of parameters being optimized and $\mathcal{M}$ specifies the amount of memory, in megabytes (considering 8 bytes to represent a single number), required for the storage of a single Hessian matrix.

Table 1: Settings for the FRM subfilters and overall prototype

parameter	Direct-form	FRM
N_b	–	157
N_1	–	383
N_p	20479	20479
L	–	128
$\mathcal{N}$	10240 (*)	271
$\mathcal{M}$	≈ 838.9 (**)	≈ 0.6

(*) 5120 unknowns are necessary if the designed prototype provides the perfect-reconstruction (PR), that is, both ISI and ICI are zero.

(**) If the problem is PR, the size of the Hessian matrix reduces to 209.7 megabytes.

For minimizing the LS objective function as given by equation (18), an SQP optimization algorithm was used, based on the `fmincon` MATLAB® command [15]. A reasonable number of points in the discrete frequency grid was found to be $10N_p$. The nonlinear equality constraints were the ones given by equation (5), in conjunction with equations (3) and (4).

The resulting magnitude response of the ISI-free FRM prototype filter is depicted in Figure 2. The magnitude responses for the corresponding L-interpolated base and masking filters are depicted in Figure 3. The first 4 out of 512 CMFB subbands are then shown in Figure 4. The figures of merit, that is, the ISI, ICI, stopband energy (E_2), and maximum stopband ripple (E_∞) are presented in Table 2 for both the direct-form PR and the FRM optimized designs. The direct-form PR design was chosen for comparisons because it has half the number of parameters of a near-PR design, thereby simplifying the optimization task. No problems were found during the optimization and the Nyquist constraints were attained with a discrepancy of order 10^{-18}.

Table 2: Resulting figures of merit for direct-form PR and ISI-free CMFB designs.

figure of merit	Direct-form	FRM
ISI	0	0
ICI (dB)	$-\infty$	-70.3
E_2	2.62×10^{-10}	1.34×10^{-09}
E_∞ (dB)	-58.4	-65.2

5. CONCLUDING REMARKS

This paper presented time-domain constraints for the design of lengthy prototype filters for CMFB and MDFTB systems. Since filter banks with a large number of subbands are of major interest, it was necessary to map the Nyquist constraints on an FRM-based prototype filter to generate an ISI-free TMUX system with reduced

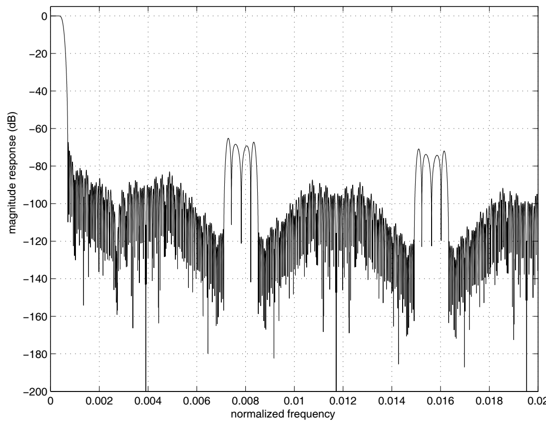

Figure 2: Magnitude response (detail) for the ISI-free FRM optimized prototype filter.

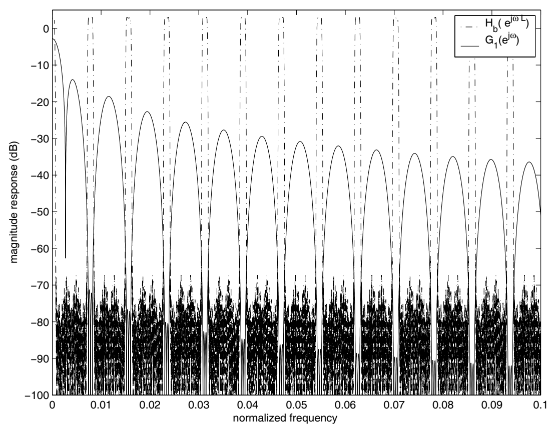

Figure 3: Magnitude responses (detail) for the FRM optimized interpolated base (dot-dashed) and masking (solid) subfilters.

number of parameters to be optimized. The problem involved nonlinear equality constraints and was easily solved using an SQP algorithm. The results showed that filter banks with very good figures of merit are attained when compared to their PR direct-form counterparts. Moreover, the required amount of physical memory to run the optimization is impressively lower than the one required by the direct-form counterpart. It is also less time-consuming than an optimization carried out in the frequency-domain.

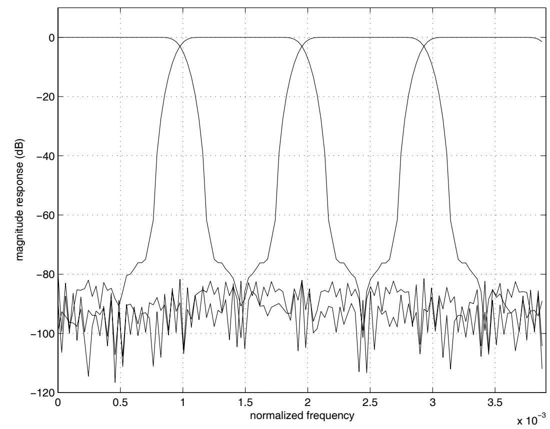

Figure 4: Magnitude response (detail) of the first four bands for the ISI-free optimized CMFB.

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