# ON A MODIFIED STRUCTURE FOR COSINE-MODULATED FILTER BANKS USING THE FREQUENCY-RESPONSE MASKING APPROACH

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**ABSTRACT**

In this paper, we propose a cosine-modulated filter bank (CMFB) design using the frequency-response masking (FRM) approach in which the interpolation factor $L$ of the base filter is approximately an even-multiple of the number $M$ of subbands. In such a case, a realizable maximally-decimated structure requires a large computational complexity for the filter bank. Instead, we employ an oversample FRM-CMFB structure to address the problem, and combine adjacent subbands to achieve the desired number of bands in the original design. The result is an FRM-CMFB structure with reduced computational complexity.

## 1. INTRODUCTION

Cosine-modulated filter banks (CMFB) utilizing frequency-response masking (FRM) method to develop its prototype filter is an efficient approach to design filter banks with large number of subbands or wide passbands with sharp transition bands [1]. There are several cases where it is possible to derive an efficient structure for the combination of the two mentioned methods, allowing the designer to implement filter banks with low computational complexity. Unfortunately, there are some cases in which the optimization of the FRM prototype filter leads to an interpolation factor $L$ which is nearly twice the number of subbands $M$. In such cases, it is not possible to implement the desired transition band, since it is located at $\omega_{3dB} \approx \pi/2M$ which is exactly the center of the repetition bands of the FRM base filter, turning the design unrealizable. We may avoid such an issue by approximating the FRM interpolation factor to a value that matches the efficient structures, that is, by forcing $L = 2K_aM + M/K_b$. Such approach, however, increases the computational complexity of the resulting structure. Alternatively, one may work with an oversampled version of the FRM-CMFB structure with $M' = 2M$ subbands, for instance, and combine adjacent channels, 2-by-2 in this case, to achieve lower computational complexity. The result is a modified TMUX structure that allows one to design FRM-CMFBs to a larger number of applications.

This paper is organized as follows: Section 2 presents the theory behind FRM filters, Section 3 introduces the transmultiplexer (TMUX) application, and Section 4 reviews the efficient and general structure for FRM-CMFB. In Section 5, we present the innovative method of combining adjacent subbands of an oversampled TMUX structure, leading to a design of $2M$ subchannels. In Section 6, a design example is shown, illustrating the advantages of the proposed design method over traditional ones. Section 7 presents concluding remarks for the paper.

## 2. FREQUENCY-RESPONSE MASKING APPROACH

The FRM approach uses four component filters in a structure shown in Figure 1. The filter $H_{b2}(z)$ is the complementary version of $H_{b1(z)}$ and both can be realized in the same structure, such that only three component filters should be designed. In this scheme, the so-called interpolated base filter $H_{b1}(z^L)$ presents a repetitive frequency spectrum which is processed by the positive masking filter $G_1(z)$ in the upper branch of the realization. Similarly, a complementary version of the repetitive frequency response, $H_{b2}(z^L)$, is filtered by the negative masking filter $G_2(z)$ in the lower branch of the realization. In this procedure, both masking filters keep some of the spectrum repetitions within the desired passband which are then added together to compose the desired overall frequency response. The magnitude responses of the filter composing this sequence of operations are depicted in Figure 2, where one can clearly see the resulting filter with very sharp transition band.

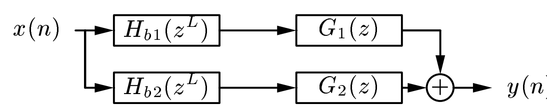



**Fig. 1**. The positive (upper) and the negative (lower) branches of the FRM filter.

If the base filter, which is interpolated, has linear-phase and an even order $N_b$, its direct and complementary transfer functions are given by

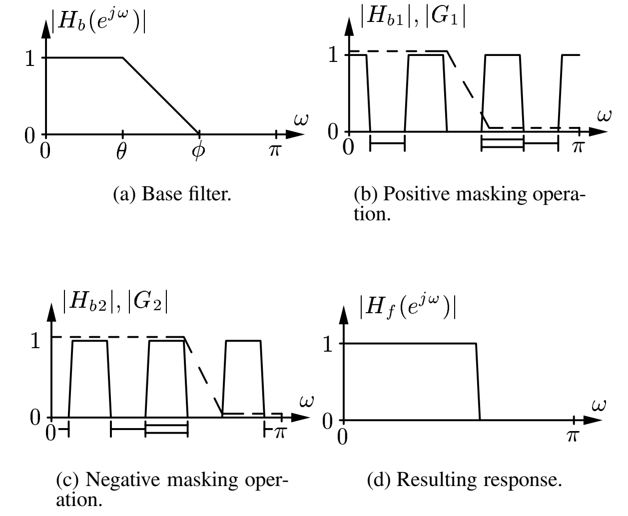
$$H_{b1}(z^L) = \sum_{i=0}^{N_b} h_b(i) z^{-Li} \tag{1}$$

$$H_{b2}(z^L) = z^{-N_b L/2} - \sum_{i=0}^{N_b} h_b(i) z^{-Li} \tag{2}$$

respectively, where $L$ is the interpolation factor and $h_b(n)$ is the impulse response of the base filter. From the equations above, we can readily see that

$$|H_{b2}(e^{j\omega})| = 1 - |H_{b1}(e^{j\omega})| \tag{3}$$

and also that $|H_{b2}(e^{j\omega})|$ can be obtained by subtracting $|H_{b1}(e^{j\omega})|$ from the signal at the central node in $H_{b1}(z)$. The cutoff frequencies $\theta$ and $\phi$ of the base filter (see Figure 2) depend on $L$ and on the desired band-edge frequencies $\omega_p$ and $\omega_s$ of the overall filter. The masking filters are simple FIR filters with band-edge frequencies that also depend on $L$ and on the bands of the interpolated filter. Therefore the optimal value of $L$ that minimizes the overall number of multiplications can be obtained by estimating the




(a) Base filter. (b) Positive masking operation. (c) Negative masking operation. (d) Resulting response.

**Fig. 2**. Frequency-response masking approach, showing the don't care bands (single line) and the critical bands (double lines below the frequency axis).

lengths of all sub-filters for various $L$ and finding the best case scenario heuristically. If the transition band is not too sharp when compared to the passband (i.e., for the narrowband design case), then it is possible to discard the lower branch of the FRM filter, reducing further the number of coefficients in the filter. Also, the specifications for the subfilters can be relaxed, since there is no overlap between the frequency-response of the two branches [2]. The narrowband case is more common when designing CMFBs, but it depends on the roll-off factor and the required attenuation.

## 3. THE TMUX CONFIGURATION

The transmultiplexer (TMUX) can be implemented using a CMFB-like structure, where the signals coming from various sources are interpolated, filtered by the synthesis filters, and added together to compose a single signal that is transmitted over a single channel $C$ [3], [4]. Once the signal is received, the analysis filters split this signal into $M$ channels, where in each channel output we have an estimated version of the input sources. Figure 3 depicts the block diagram for such system.

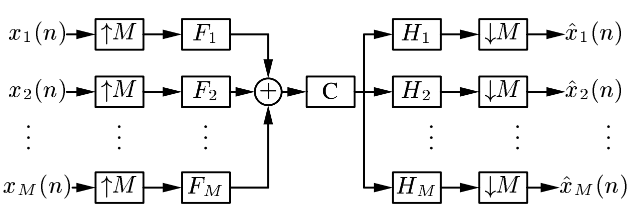



**Fig. 3**. The block diagram for an $M$-channel TMUX.

In a noiseless environment and if the channel is a delay, then the output signals are equal to the source signals if the TMUX has perfect reconstruction (PR). Otherwise, if the transmitted and received signals are very close, we have the nearly-prefect reconstruction (NPR) case The main advantage of using a CMFB is the fact that only the prototype filter design is needed [3]. Once this filter is designed, the synthesis and the analysis filters can be obtained by modulating the prototype filter with a proper cosine function. The prototype filter of order $N_p$ is of the form

$$H_p(z) = \sum_{n=0}^{N_p} h_p(n) z^{-n}, \quad h_p(N_p - n) = h_p(n) \tag{4}$$

The cutoff frequencies can be determined by using the roll-off factor and the 3dB point of the amplitude response, that must be located approximately at $\omega = \pi/(2M)$. The roll-off factor gives us the stopband edge

$$\omega_s = \frac{(1+\rho)\pi}{2M} \tag{5}$$

The analysis and the synthesis filters are given respectively by

$$h_k(n) = 2h_p(n)\cos\left(\frac{(2k+1)(n - N_p/2)\pi}{2M} + (-1)^k \frac{\pi}{4}\right) \tag{6}$$

and

$$f_k(n) = 2h_p(n)\cos\left(\frac{(2k+1)(n - N_p/2)\pi}{2M} - (-1)^k \frac{\pi}{4}\right) \tag{7}$$

for $k = 0, 1, \ldots, M-1$. The inter-symbol interference (ISI) and the inter-carrier interference (ICI) are used to measure the TMUX performance and are definied as [4]

$$\text{ISI} = \max_k \left( \sum_n (\delta(n) - t_k(n))^2 \right) \tag{8}$$

$$\text{ICI} = \max_k \left( \sum_{l=0, k \neq l}^{M-1} |[\mathbf{T}(e^{j\omega})]_{kl}|^2 \right) \tag{9}$$

where $\delta(n)$ is the ideal impulse, $t_k(n)$ is the impulse response for the $k$th channel output, and the term $[\mathbf{T}(e^{j\omega})]_{kl}$ is the crosstalk term [4].

## 4. FRM-CMFB DESIGN

By using only the positive branch of the FRM structure of Figure 1 as the prototype filter for the CMFB, the transfer function for the analysis filters becomes

$$H_m(z) = \sum_{n=0}^{N} c_{m,n} (h_{b1}^I * g_1)(n) z^{-n} \tag{10}$$

where $c_{m,n}$ is the cosine function as it appears in Eq. (6), the term $(h_{b1}^I * g_1)(n)$ denotes the convolution between the interpolated base filter and the positive masking filter responses, and $N$ is the overall order of the FRM filter. Thus, the key point is to find an efficient structure that evaluates the convolution in Eq. (10), by taking into consideration the proper cosine functions for each sample. For $L = \frac{M}{K}, K = 1, 2, \ldots$, and after some manipulations, Eq. (10) can be written as

$$H_m(z) = \sum_{q=0}^{Q-1} \left\{ z^{-Lq} H_{b1q}(-z^{2M}) \sum_{j=0}^{2M-1} c_{m,j+Lq} z^{-j} E_j(-z^{2M}) \right\} \tag{11}$$

where $H_{b1q}(z)$ and $E_j(z)$ are the polyphase decompositions of the base filter and of the positive masking filter, respectively, and

$Q = \frac{2M}{L}$ is the number of polyphase components required for the base filter. This result leads to the structure depicted in Figure 4. In the general FRM structure, the same analysis can be made for the lower branch, and the responses of the two branches should be combined just before the modulating stage.

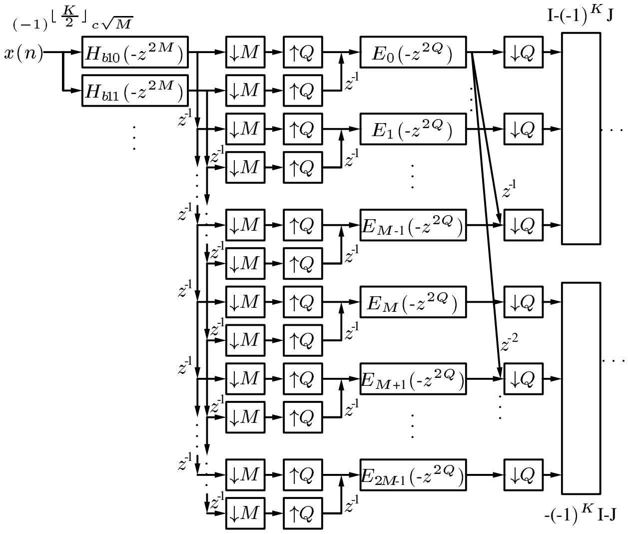



**Fig. 4**. The realization of the FRM in a CMFB structure that precedes the DCT-IV operation.

As we can see from Figure 4, the base filter will have $M_b = K_a Q$ coefficients and the masking filter will have $M_m = 2K_b M$ coefficients in order to perform the polyphase decompositions.

## 5. OVERSAMPLED FRM-CMFB STRUCTURE

Instead of using the standard TMUX structure of Figure 3, it is possible to use a synthesis bank with $M' = 2M$ subchannels, and then combine its adjacent branches 2-by-2 to derive the originally desired $M$ subchannels. Figure 5 depicts this new TMUX structure. Thus, in this structure, each subband has the same width $\pi/M$ as desired, but now, from the design point of view, we have a $2M$-channel TMUX with interpolation factor $M$. This indicates that we employ an oversampled TMUX instead of a maximally interpolated TMUX structure. In fact, in this structure, $x_1(n)$ only needs to be interpolated by $M$, because it will be filtered by $F_1'$ and $F_2'$, which effectively splits the spectrum of $x_1$ into two bands of width $\pi/M' = \pi/2M$ each. The same process occurs to the various inputs $x_i(n)$. In this proposed structure, one can interpret that signal $x_1(n)$ is represented by two components, one with the lower frequencies and with for the higher frequencies, which are the outputs of the filters $F_1'$ and $F_2'$, respectively. Analogously, for the analysis filter bank, signal $\hat{x}_1(n)$ is obtained by adding the responses of the filters $H_1'$ and $H_2'$ which filter the lower and the higher frequencies of the signal $x_1(n)$, respectively.

Hence, with this new TMUX structure, it is possible to use the general structure of FRM-CMFB designs with $L = kM'$, including the $L = 2kM$ case which previously lead to an unrealizable structure, as discussed in [1]. Also, it is important to notice that the channel throughput is not increased, as we are working with an oversampled filter bank. Moreover, since we are adding each two adjacent channels, the roll-off factor of the modified $2M$-channel structure can be relaxed to twice the value for the original $M$-band

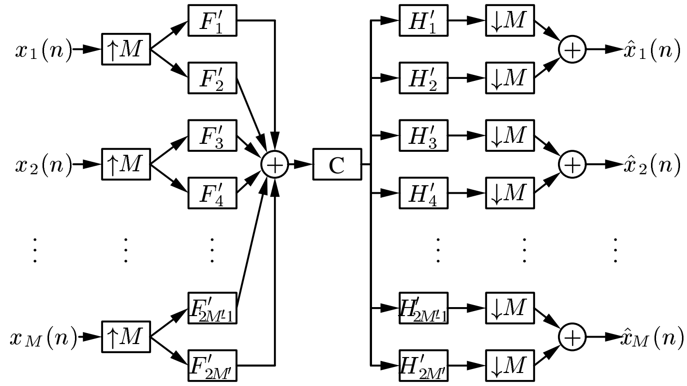



**Fig. 5**. The block diagram for a $M$-channel TMUX as sum of adjacent channel.

case. If PR is to be achieved, then the sum of adjacent channels will also present the PR property. If only NPR is required, the sum of the adjacent channels introduces little distortion, which will also tend to present NPR characteristics, as illustrated below.

## 6. NUMERICAL EXAMPLE

Consider the design of a $(M = 8)$-channel filter bank, with passband ripple of 0.1 dB, minimum stopband atenuation of 80 dB, and a roll-off factor of $\rho = 0.025$. In this case, by running an order-estimating algorithm (e.g. remezord) for the FRM composing filters, the parameters shown in Table 1 are obtained.

**Table 1**. FRM characteristics for $M = 8$ channels ($\rho = 0.025$).

| $L$ | $N_b$ | $N+$ | $N-$ | $\mathcal{M}_{tot}$ |
|---|---|---|---|---|
| 6 | 519 | 35 | 0 | 554 |
| 7 | 447 | 45 | 0 | 492 |
| 8 | 391 | 57 | 0 | 448 |
| 9 | 347 | 71 | 0 | 418 |
| 10 | 313 | 93 | 0 | 406 |
| 11 | 285 | 120 | 0 | 405 |
| 12 | 261 | 163 | 0 | 424 |
| 13 | 241 | 234 | 0 | 475 |
| 14 | 225 | 376 | 0 | 601 |
| 15 | 211 | 801 | 0 | 1012 |
| 16 | Inf | 0 | 0 | Inf |
| 17 | 185 | 908 | 64 | 1157 |
| 18 | 175 | 482 | 72 | 729 |
| 19 | 167 | 340 | 82 | 589 |
| 20 | 159 | 269 | 93 | 521 |
| 21 | 151 | 227 | 105 | 483 |
| 22 | 145 | 198 | 120 | 463 |
| 23 | 139 | 178 | 140 | 457 |
| 24 | 133 | 163 | 163 | 459 |
| 25 | 127 | 151 | 193 | 471 |
| 26 | 123 | 142 | 234 | 499 |

From this table, it is possible to note that the interpolating factor that minimizes the total number of coefficients for the FRM filter is $L = 11$. Unfortunately, this value of $L$ does not lead to any of the efficient FRM-CMFB structures described in [1]. Two alternative designs are to employ $L = M/K_b = 8$, with $K_b = 1$, or $L = 2K_a M + M/K_b = 24$, with $K_a = 1$ and $K_b$=1. In both cases, however, the computational complexity increases at least

10% of its orginal value for $L = 11$. Another possibly good alternative is to employ $L = 2K_aM = 16$, with $K_a = 1$, which is unrealizable, as described in [1]. Now, by using the modified TMUX structure of Section 5, an $(M' = 16)$-channel design can be performed, readily adjusting the roll-off factor to 0.05. The FRM characteristics for this new set of specifications are summarized in Table 2.

**Table 2**. FRM characteristics for $M' = 16$ channels ($\rho = 0.05$).

| $L$ | $N_b$ | $N+$ | $N-$ | $\mathcal{M}_{tot}$ |
|---|---|---|---|---|
| 6 | 519 | 26 | 0 | 545 |
| 7 | 447 | 32 | 0 | 479 |
| 8 | 391 | 38 | 0 | 429 |
| 9 | 347 | 44 | 0 | 391 |
| 10 | 313 | 52 | 0 | 365 |
| 11 | 285 | 58 | 0 | 343 |
| 12 | 261 | 68 | 0 | 329 |
| 13 | 241 | 76 | 0 | 317 |
| 14 | 225 | 86 | 0 | 311 |
| 15 | 211 | 98 | 0 | 309 |
| 16 | 197 | 110 | 0 | 307 |
| 17 | 185 | 124 | 0 | 309 |
| 18 | 175 | 140 | 0 | 315 |
| 19 | 167 | 160 | 0 | 327 |
| 20 | 159 | 182 | 0 | 341 |
| 21 | 151 | 206 | 0 | 357 |
| 22 | 145 | 237 | 0 | 382 |
| 23 | 139 | 275 | 0 | 414 |
| 24 | 133 | 322 | 0 | 455 |
| 25 | 127 | 383 | 0 | 510 |
| 26 | 123 | 464 | 0 | 587 |

From this new table, it is possible to note that with the modified TMUX structure, the interpolation factor of $L' = 16$ achieves the minimum computational complexity, which is approximately 20than the case of $L = 11$ for $M = 8$ channels. Also, for this new design it is possible to use the efficient structure associated to the case $L = M'/K_b = 16$, with $K_b = 1$. The only difference is that we do not interpolate the signals by 16, as we interpolated them by 8 and add each two adjacent channels.

Figure 6 shows the magnitude response overall filter bank for the 8-channel standard FRM-CMFB design with $L = 8$. Figure 7 shows the magnitude responses 16-channel filter bank for the modified FRM-CMFB design with $L = 16$, before (a) and after (b) combining each two adjacent channels. The ISI and ICI values for the three designs are shown in table 3. These results illustrate how the proposed method may also improve the performance of the designed FRM-CMFB structure with respect to such practical figures of merit.

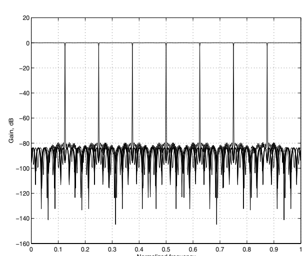



**Fig. 6**. Magnitude responses of 8-channel FRM-CMFB with standard TMUX and $L = 8$.

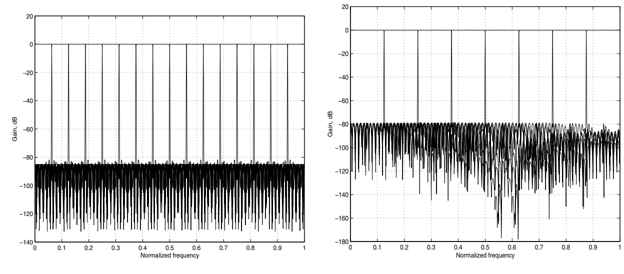



**Fig. 7**. Magnitude responses of 16-channel FRM-CMFB with $L = 16$: (a) auxiliary 16-channel filter bank;(b) desired 8-channel filter bank.

**Table 3**. ISI and ICI values for the designed filter banks.

| Value | 8-channel | 16-channel | 8-channel adjacent added |
|---|---|---|---|
| ISI | -35 dB | -40 dB | -40 dB |
| ICI | -75 dB | -85 dB | -75 dB |

## 7. CONCLUDING REMARKS

The FRM-CMFB represents an efficient structure for designing filter banks and transmultiplexers (TMUXes). In this paper, we investigated the design of FRM-CMFBs when the FRM interpolation factor $L$ is an even multiple of the number $M$ of subchannels. For such a case, a modified oversampled TMUX structure is proposed allowing one to design FRM-CMFBs with less constraints on the relationship between $L$ and $M$. With such modified design procedure one can take advantage of the efficient FRM-CMFB structure in a much larger number of applications. A design example is included illustrating the improvements achieved by the proposed method.

## 8. REFERENCES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