On the Design of Cosine-Modulated Filter Banks Using Recurrent Frequency-Response Masking

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***Abstract*— The cosine-modulated filter bank (CMFB) is an attractive filter bank structure with a simplified design, being based on a single prototype filter. Recent advances have reduced the overall CMFB computational cost by employing the frequency-response masking (FRM) approach to design the prototype filter. The present paper discusses the use of recurrent-FRM stages to design CMFBs. It is verified how by applying several layers of the FRM method can further reduce the computational complexity of resulting CMFB structures. This technique also yields a simpler procedure when one is interested on optimizing the CMFB performance with respect to practical figures-of-merit such as the intersymbol interference (ISI) and interchannel interference (ICI).**

I. Introduction

Cosine-modulated filter banks (CMFBs) are a commonly used tool in signal processing [1], [2], [3], whose main advantages include: simple design since a single prototype filter is required, and computationally efficient implementation.

The frequency-response masking (FRM) approach is a design technique for FIR digital filters, particularly with narrow transition bands [4], [5], with reduced computational complexity.

In this paper, we analyze the use of the FRM approach to design the prototype filter of a CMFB. The recurrent use of the FRM method to design the FRM elementary filters is investigated. The result is further reduction in the computational complexity of the CMFB implementation, when compared to the FRM-CMFB, and the possibility of designing filter banks with extremely high number of bands.

The organization of this paper is as follows: In Section 2 and Section 3, we describe the basic concepts behind the standard CMFB and FRM methods, respectively. In Section 4, the FRM-CMFB structure is revised, and in Section 5, the recurrent-FRM CMFB structure is presented. Design examples are included in Section 6 and Section 7, illustrating the results achieved with the proposed method.

II. The CMFB structure

A CMFB prototype filter is described by a 3-dB attenuation point and the stopband edge frequency is given by [6], [7]

$$\omega_{3\text{dB}} \approx \pi/(2M); \quad \omega_r = \frac{(1+\rho)\pi}{2M} \tag{1}$$

where the roll-off factor ρ controls the overlapping between adjacent bands. If the prototype filter has order N_p and transfer function

$$H_p(z) = \sum_{n=0}^{N_p} h_p(n) z^{-n} \tag{2}$$

then the impulse responses of the analysis and the synthesis filters are given by

$$h_m(n) = 2h_p(n)\cos\left[\frac{(2m+1)(n-N_p/2)\pi}{2M} + (-1)^m\frac{\pi}{4}\right] \tag{3}$$

$$f_m(n) = 2h_p(n)\cos\left[\frac{(2m+1)(n-N_p/2)\pi}{2M} - (-1)^m\frac{\pi}{4}\right] \tag{4}$$

respectively, for $m = 0, 1, \ldots, (M-1)$, and $n = 0, 1, \ldots, N_p$.

If the CMFB prototype filter has $(N_p+1) = 2KM$ coefficients, then one can perform a $2M$-polyphase decomposition on $H_p(z)$ such that

$$H_p(z) = \sum_{j=0}^{2M-1} z^{-j} E_j(z^{2M}) \tag{5}$$

with

$$E_j(z) = \sum_{k=0}^{K-1} h_p(2kM + j) z^{-k} \tag{6}$$

for $j = 1, \ldots, 2M$.

After standard algebraic manipulations [3], the analysis filters can then be described by

$$\begin{aligned} H_m(z) &= \sum_{j=0}^{2M-1}\left[c_{m,j} z^{-j} \sum_{k=0}^{K-1} (-1)^k h_p(2kM+j) z^{-2kM}\right] \\ &= \sum_{j=0}^{2M-1} c_{m,j} z^{-j} E_j(-z^{2M}) \end{aligned} \tag{7}$$

for $m = 0, 1, \ldots, (M-1)$, where $c_{m,j}$ is the cosine function as it appears in Eq. (3). Based on such description, the analysis filter bank can be efficiently implemented as described in [3]. A similar reasoning can be employed for the synthesis filter banks as well.

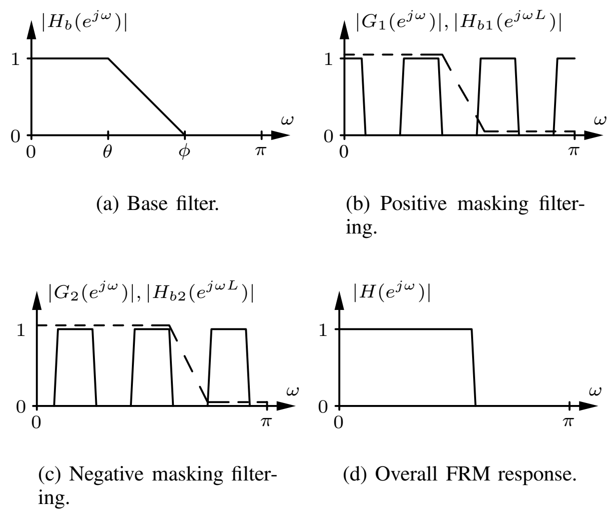
III. The FRM method

In such an approach, the desired frequency response is formed by an interpolated base filter $H_{b1}(z^L)$, with repetitive spectrum, and its complementary response $H_{b2}(z^L)$. These filters are followed by the corresponding masking filters,

$G_1(z)$ and $G_2(z)$, which eliminate the undesired spectrum repetitions, and whose outputs are added together to compose the desired passband while taking into account the transition band. In that framework, the FRM filter can be described by [4], [5]

$$H(z) = H_{b1}(z^L)G_1(z) + H_{b2}(z^L)G_2(z) \tag{8}$$

This sequence of operations is depicted in Figure 1, which clearly depicts the resulting sharp transition band of the FRM filter. For narrowband designs, the FRM lower branch can be eliminated further reducing the resulting computational complexity.



(a) Base filter.

(b) Positive masking filtering.

(c) Negative masking filtering.

(d) Overall FRM response.

Fig. 1. Magnitude responses in FRM method.

IV. The FRM-CMFB structure

By using only the positive branch of the FRM structure as the prototype filter for the CMFB, the transfer function for the analysis filters becomes

$$H_m(z) = \sum_{n=0}^{N} c_{m,n}(h_{b1}^I * g_1)(n)z^{-n} \tag{9}$$

where $c_{m,n}$ is the cosine function of Eq. (3), the term $(h_{b1}^I * g_1)(n)$ denotes the convolution between the interpolated base filter and the positive masking filter responses, and N is the overall order of the FRM filter.

Assuming that $H_{b1}(z)$ and $G_1(z)$ have orders N_b and N_m, respectively, and using the definition of convolution, Eq. (9) can be rewritten as

$$H_m(z) = \sum_{i=0}^{N_b}\left[h_b(i)z^{-Li}\sum_{n=0}^{N_m} c_{m,(n+Li)}g_1(n)z^{-n}\right] \tag{10}$$

If the interpolation factor can be written as

$$L = 2K_aM + \frac{M}{K_b} \tag{11}$$

where $K_a \geq 0$ and $K_b > 0$ are integer numbers, then the analysis filters can be expressed as

$$H_m(z) = \sum_{q=0}^{Q-1}\Bigg[z^{-Lq}H'_{b1q}(-z^{LQ}) \times \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K_b}q)}z^{-j}E'_j(-z^{2M})\Bigg] \tag{12}$$

where

$$H'_{b1q}(z) = \sum_{k=0}^{K_c-1}(-1)^{K_aq}h_b(kQ+q)z^{-k} \tag{13}$$

for $q = 0, 1, \ldots, (Q-1)$, where $Q = 2K_b$ and $(N_b+1) = QK_c$, and an efficient FRM-CMFB structure results [8], [9].

V. Recurrent-FRM CMFB structure

The computational complexity of the FRM-CMFB structure can be further reduced if one of the elementary FRM filters is designed with the FRM method recurrently. This can be performed by using a recurrent-FRM base filter or a recurrent-FRM masking filter, leading to the two FRM structures depicted in Figures 2 and 3, respectively.

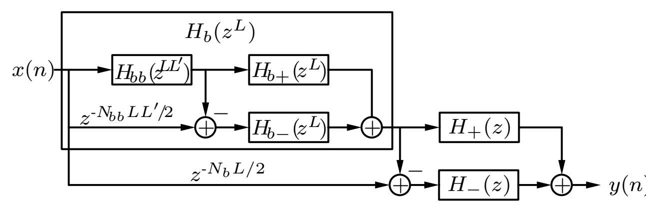


Fig. 2. FRM filter with recurrent base filter.

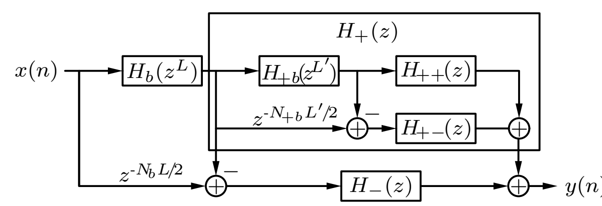


Fig. 3. FRM filter with recurrent positive-masking filter.

Considering the recurrent-FRM method applied to the masking filter, and disregarding the negative branch to simplify the analysis, one can write that

$$H_m(z) = \sum_{n=0}^{N} c_{m,n}((h_{b1}^I * h_b^{I'}) * g_1)(n)z^{-n} \tag{14}$$

where $(h_{b1}^I * h_b^{I'})(n)$ denotes the convolution of the two interpolated base filters. Notice, however, that this convolution must present an overall interpolation factor that satisfies the restriction in equation (11). For instance, if L is a multiple of L', we can write that

$$H_m(z) = \sum_{i=0}^{N_B}\left[(h_{b1}^{I_2} * h'_b)(i)z^{-L'i}\sum_{n=0}^{N_m} c_{m,(n+L'i)}g_1(n)z^{-n}\right] \tag{15}$$

where N_B is the order of the convolution $(h_{b1}^{I} * h_b^{I'})(n)$, and $h_{b1}^{I_2}$ represents the original base filter interpolated by a factor of L/L'. From equation (15), the values of $c_{m,(n+L'i)}$ depend only on L', and therefore the two base filters together will not misalign the DCT-IV terms in the masking filter decomposition. One can then rewrite $H_m(z)$

$$H_m(z) = H_{b1}(-z^L) \sum_{q=0}^{Q'-1} \Bigg[z^{-L'q} H'_{bq}(-z^{L'Q'}) \times \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K'_b}q)} z^{-j} E'_j(-z^{2M}) \Bigg] \quad (16)$$

where Q', L', and K'_b are the respective counterparts of Q, L, and K_b, in the polyphase decomposition of the second base filter, $H'_b(z)$. The building block $\bar{H}_{b1q}(-z^{L'Q'})$ represents the z transform of the convolutions between $h_{b1}^{I_2}$ and each polyphase component of $H'_b(z)$, the second base filter. From this equation, one notices that in the recurrent-FRM CMFB structure, it is then necessary to decompose the second-stage base filter by a factor of $Q' = 2M/L'$, and to introduce a slightly changed version of the interpolated base filter, $H_{b1}(-z^L)$, at the input.

VI. Recurrent FRM design

Example 1: To illustrate the possible benefits from using the recurrent FRM method, consider the filter design whose specifications are given in Table I.

TABLE I

Normalized digital filter specifications in Example 1.

Passband edge:	$\omega_p = 0.2\pi$
Stopband edge:	$\omega_r = 0.201\pi$
Passband ripple:	$A_p = 0.1$ dB
Stopband attenuation:	$A_r = 80$ dB

In order to satisfy these specifications, an equiripple linear-phase FIR filter would require an order $N = 9517$, whereas a standard FRM filter can be designed, as described in the Design 1 entry in Table II, requiring only $M = 725$ non-zero coefficients. In Table II the parameters shown represent interpolation factors (Ls), base and masking filter orders (Ns), and the number of multiplications (Ms) on some building blocks.

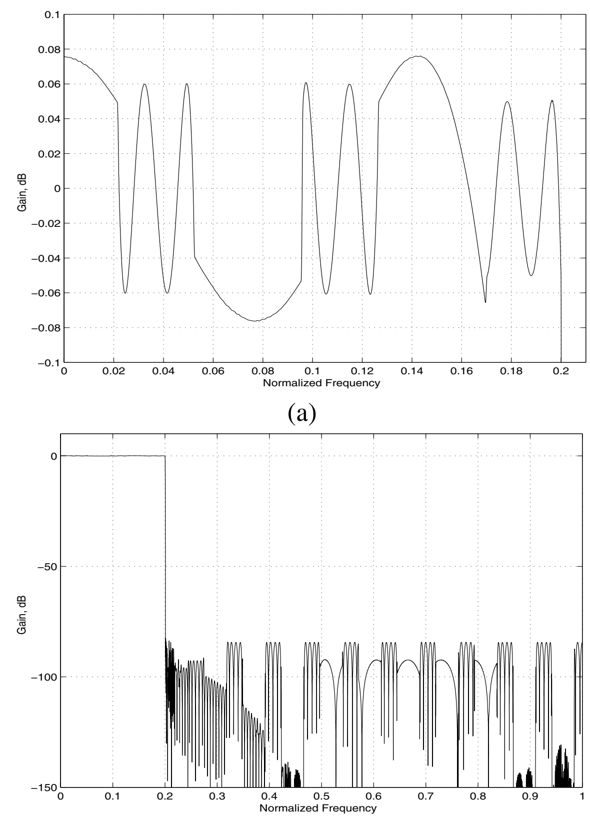
TABLE II

FRM filter characteristics in Example 1.

	L	N_b	N_+	N_-	M	N
Design 1	27	356	214	152	725	9826
	L_b	N_{bb}	N_{b+}	N_{b-}	M_b	N'_b
Design 2	4	90	32	58	183	418
	L_+	N_{+b}	N_{++}	N_{+-}	M_+	N'_+
Design 3	3	120	24	-	146	384

The FRM design can be further simplified if one employs the FRM method recurrently to design the base and masking filters. Following this strategy, one obtains the Design 2 and Design 3 entries in Table II. Design 2 indicates that the base filter can require only $M_b = 183$ non-zero coefficients as opposed to 357 in the standard FRM design. Meanwhile, in Design 3 the original FRM masking filter is designed with $M_+ = 146$ non-zero coefficients instead of 215 in the standard FRM design.

Combining all designs, the recurrent-FRM filter presents an equivalent order of $N = (N'_b \times L + N'_+) = 11679$, requiring solely $M = [M_b + M_+ + (N_- + 1)] = 482$ non-zero coefficients. Figure 4 depicts the magnitude response obtained by combining the results of Design 2 and 3.



(b)

Fig. 4. Magnitude response of recurrent-FRM filter in Example 1: (a) Passband detail; (b) Complete response.

VII. Recurrent-FRM CMFB design

Consider the design of a CMFB with $M = 1024$ channels, $\rho = 0.1$, $A_p = 0.2dB$, and $A_r = 60dB$, leading to the specifications

$$\omega_p = (1 - \rho + 0.037)\pi/2048 \quad (17)$$

$$\omega_r = (1 + \rho)\pi/2048 \quad (18)$$

where 0.037 accounts for a reduction in the transition band obtained by the resulting design. A standard FIR design is not possible since it would require an extremely high order.

Some FRM scenarios are given in Table III, where one can see that the optimal design (with respect to minimum number of non-zero coefficients) has $L = 169$, which does not satisfy equation (11). Possible options are $L = 256$ and $L = 1024$, associated to Design 4 and Design 5, respectively, in Table IV.

TABLE III
STANDARD FRM SCENARIOS IN EXAMPLE 2.

L	N_b	N_+	$\mathcal{M}_{\text{tot}}$	N_{equiv}	Q
1	87462	0	87463	87462	2048
2	43732	3	43737	87467	1024
4	21868	11	21881	87483	512
8	10936	23	10961	87511	256
16	5470	47	5519	87567	128
32	2736	91	2829	87643	64
64	1370	183	1555	87863	32
128	686	375	1063	88183	16
169	520	505	1027	88385	-
256	**344**	**801**	**1147**	**88865**	**8**
512	174	1873	2049	90961	4
1024	**88**	**5683**	**5773**	**95795**	**2**

Applying the recurrent-FRM method to design the Design 4 masking filter, in an attempt to reduce the number of $M = 1147$ non-zero coefficients, we obtain the Design 6 characteristics also included in Table IV. In such scenario the number of non-zero coefficients for the masking filter is reduced from 802 to only $M_+ = 117$.

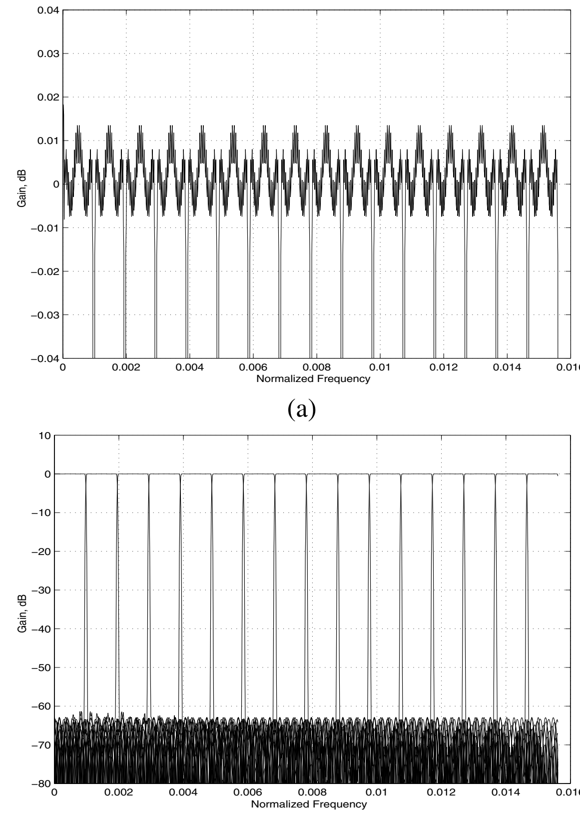
Applying the recurrent-FRM method to design the Design 5 masking filter with $M = 5773$ non-zero coefficients, we obtain the Design 7 described in Table IV. In such a case the number of non-zero coefficients for the masking filter is reduced to only $M_+ = 307$. The resulting CMFB in this case is depicted in Figure 5.

TABLE IV
FRM FILTER CHARACTERISTICS IN EXAMPLE 2.

	L	N_b	N_+	M	N	Q
Design 4	256	344	801	1147	88365	8
Design 5	1024	88	5683	5773	95795	2
	L_+	N_{+b}	N_{++}	M_+	N'_+	
Design 6	16	66	49	117	1105	
Design 7	64	116	189	307	7613	

VIII. CONCLUSION

It was shown how the frequency-response masking (FRM) approach can be applied for designing the prototype filter in cosine-modulated filter banks (CMFBs). The recurrent use of the FRM method for designing the CMFB prototype filter was investigated. It was verified that the recurrent-FRM method can further reduce the overall computational complexity of the resulting CMFB structure.



(a)

(b)

Fig. 5. Magnitude response of first 16 bands of recurrent-FRM CMFB in Example 2: (a) Passband detail; (b) Overall response.

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