Statistical Modeling of the Average Channel Gain and Delay Spread in In-Home PLC Channels

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***Abstract*—This work describes a complete statistical modeling of the average channel gain in dB (ACG$_{dB}$) and the root mean squared delay spread (RMS-DS) for power line communication (PLC) systems. The PLC channel features are estimated from $148{,}037$ channel frequency responses measured in 7 typical different places in an urban area in Brazil. Two frequency bands are considered: from 1.7 up to 30 MHz and from 1.7 up to 100 MHz. The resulting datasets for ACG$_{dB}$ and RMS-DS were fitted to well known continuous distributions, including symmetric (Logistic and Normal) and asymmetric (Exponential, Gamma, Inverse Gaussian, Loglogistic, Lognormal, Nakagami, Rayleigh, Rician, Skew-normal, t-Student and Weibull) cases. The best distribution fitted to the considered dataset is indicated by the log-likelihood value and three distinct information criteria. The achieved results revealed that the ACG$_{dB}$ is better modeled by the Skew-normal and the Nakagami distributions for the frequency bands from 1.7 up to 30 MHz and 100 MHz, respectively, whereas the RMS-DS is little bit better modeled by the Gamma distribution, then by the Lognormal distribution, in both frequency bands considered.**

Keywords*—Statistical modeling, delay spread, average channel gain, maximum likelihood.

I. INTRODUCTION

Power line communication (PLC) is the technology that makes use of the existing and ubiquitous electrical infrastructure to provide data communication. This technology has received considerable attention due to its reduced installation costs, as most needed infrastructure (power line cables) is already installed. On the other hand, since electric power grids were not initially designed for data communications purposes, they represent a challenging transmission medium. Therefore, the success of PLC systems requires a thorough study of the electric power grids main features that affect data communication.

Among the features of interest the average channel gain (ACG) and the root mean squared delay spread (RMS-DS) stand out, and their normality/lognormality is discussed in a few works on the related literature. In [1], for instance, measurements of the PLC channel in an urban and suburban areas in US are presented, considering a frequency band up to 30 MHz. In [2], normality tests based on 60 PLC channel estimates taken in six different homes did not reject the null hypothesis in which the ACG$_{dB}$ (ACG in dB) and the RMS-DS were considered normal and lognormal variables, respectively. The results of 200 PLC channels measured in 25 different premises in Spain are reported in [3], using a 30 MHz frequency band. In that work, the normality assumption for ACG$_{dB}$ was rejected by all performed tests, whereas the lognormality assumption was validated for the delay spread metric. Finally, in Italy, a set of 1266 channels were measured considering a frequency band up to 100 MHz [4], and the normality of ACG$_{dB}$ was not strictly confirmed, whereas the RMS-DS lognormality was firmly established.

In this work, the statistical modeling of the ACG$_{dB}$ and RMS-DS features is performed using the PLC channel estimates from 7 typical different places in an urban area in Brazil. Two frequency bands are considered: Band A (from 1.7 up to 30 MHz) and Band B (from 1.7 up to 100 MHz). The resulting datasets are then modeled by several statistical distributions, including symmetric (Logistic and Normal) and asymmetric (Exponential, Gamma, Inverse Gaussian, Loglogistic, Lognormal, Nakagami, Rayleigh, Rician, Skew-normal, t-Student and Weibull) distributions. The best distribution is evaluated by the associated log-likelihood value and three information criteria, namely the Akaike information criterion (AIC), Bayesian information criterion (BIC) and the efficient determination criterion (EDC). Results indicate that, in Band A and Band B, the ACG$_{dB}$ feature presents Skew-normal and Nakagami distributions, respectively, whereas the RMS-DS is better fitted in both frequency bands by a Gamma distribution, having the Lognormal distribution quite similar fitting.

The remainder of this work is organized as follows: Section II details the PLC channel measurements employed in the subsequent analyses and Section III presents the PLC parameters (ACB$_{dB}$ and RMS-DS) to be modeled. Section IV discusses the formal concepts behind the maximum likelihood estimation process. Section V presents the modeling results for the ACB$_{dB}$ and RMS-DS features, including a comparison to other analyses found in the related literature. Finally, Section VI closes the paper summarizing its main contributions.

II. MEASUREMENT CAMPAIGN

A measurement campaign was performed in seven different typical sites in an urban area in Brazil, as detailed in Table I. The Brazilian in-home PLC channels were considered and analyzed in two frequency bands: Band A (from 1.7 up to 30 MHz) and Band B (from 1.7 up to 100 MHz).

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Table I: Main features of the measured places.

Construction type	Age (years)	Constructed area (m^2)
House 1	30	78
House 2	10	69
Apartment 1	9	54
Apartment 2	9	42
Apartment 3	18	65
Apartment 4	3	62
Apartment 5	2	54

In the entire campaign, 245 different outlet pairs were employed, providing a total of 148,037 channel frequency response (CFR) estimates, with an average of 604 CFR estimates for each electric circuit configuration. Each CFR was obtained by using the channel-estimation methodology fully described in [5].

III. Parameters Under Investigation

This section presents the formal definitions of the ACG and RMS-DS parameters of PLC systems.

A. Average channel gain

The ACG$_{dB}$ of a PLC channel is expressed by

$$\text{ACG}_{dB} = 10\log_{10}\left(\frac{1}{B}\int_B |H(f)|df\right), \quad (1)$$

where $H(f)$ is the channel frequency response at the frequency f and B is the frequency bandwidth. The average channel attenuation (ACA), given by ACA $= -\text{ACG}_{dB}$, is adopted in this contribution, as some of the statistical distributions considered here cannot assume negative values.

B. Root mean squared delay spread

The RMS-DS denotes the distribution of the transmitted power over the various paths in a multipath environment, and can be defined as the square root of the second central moment of a power delay profile. For a channel impulse response (CIR) $h(t)$, the power delay profile can be calculated with

$$P(\sigma) = \frac{|h(t)|^2}{\int_{-\infty}^{\infty}|h(t)|^2 dt}. \quad (2)$$

The resulting RMS-DS is given by

$$\sigma_\tau = \int (\sigma - \sigma_e - \sigma_A)^2 P(\sigma)d\sigma, \quad (3)$$

where σ_A corresponds to the time delay of the first transmitted signal at the receiver and σ_e is the mean excess delay given by $\sigma_e = \int(\sigma - \sigma_A)P(\sigma)d\sigma$.

IV. Maximum Likelihood Estimation

Let $X_1,\ X_2, ..., X_n$ be a random sample of probability density function (pdf) $f(x|\boldsymbol{\theta})$, where $\boldsymbol{\theta} = [\theta_1, ..., \theta_K]^T$ represents the set of k unknown parameters. Thus, the likelihood function can be defined as [6]

$$L(\boldsymbol{\theta}) = \prod_{i=1}^{n} f(x_i|\theta_1, ..., \theta_K), \quad (4)$$

and the associated log-likelihood function is given by

$$l(\boldsymbol{\theta}) = \sum_{i=1}^{n}\log(f(x_i|\theta_1, ..., \theta_K)). \quad (5)$$

The maximum likelihood estimate (MLE), represented by the vector $\hat{\boldsymbol{\theta}}$, is obtained through

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}}\ l(\boldsymbol{\theta}). \quad (6)$$

Such optimization problem can be easily solved for some distributions, such as the normal distribution. Other distributions, however, such as, for instance, the skew-normal and gamma distributions, do not allow an analytic solution, and require a numerical procedure to determine the corresponding MLE, as detailed in [7].

V. Statistical Analyses

The statistical characterization of the ACA feature, including its maximum, minimum, mean and standard deviation values from the 148,037 CFR estimates, along with the 50^{th} and 90^{th} percentiles is presented in Table II. The percentile reflects a value below which a given percentage of observations fall.

As can be noted, the Band A ACA values present maximum, mean and minimum values of 51.0 dB, 23.3 dB and 9.1 dB, respectively. Also, in 90% of the cases, the attenuation was below 34.7 dB. Considering the same band, the PLC channels estimated in US [8], however, presented a mean ACA value of 48.9 dB, which is more than 20 dB higher than the estimated value for Brazilian in-home PLC channels.

Using Band B, our measurements resulted in maximum, mean and minimum ACA values equal to 55.3 dB, 30.2 dB and 13.6 dB, respectively. In this case, comparisons with PLC channels in Italy [4] show that the mean ACA values in Brazilian PLC channels are around 5 dB lower.

Regarding symmetry, a significant difference between the mean and the median (50^{th} percentile) values indicate that the Band A ACA can be modeled by an asymmetric statistical distribution. On the other hand, the Band B ACA shall be better fitted by a symmetric distribution, due to the small difference between its mean and median values.

Table II: ACA statistical characterization for the measured Brazilian in-home PLC channels.

Average channel attenuation (dB)		
	Band A	Band B
Maximum	51.089	55.269
Minimum	9.145	13.557
Mean	23.281	30.211
Standard Deviation	8.609	9.158
50^{th} percentile	22.767	30.822
90^{th} percentile	34.693	39.649

Regarding the RMS-DS, the estimated statistical parameters are summarized in Table III.

In this case, for the Band A, the mean RMS-DS value was around 0.15 μs, which is much lower than the corresponding US value of 0.53 μs reported in [8]. Also, while the RMS-DS is higher than 0.14 μs in 50% of the in-home PLC channels in Brazil, this number rises to 0.47 μs for the same percentage in US.

For Band B, the mean RMS-DS value was 0.13 μs. In comparison to PLC channels in France [9], Brazilian in-home PLC channels presented lower RMS-DS values. In fact, while the RMS-DS can reach values of 0.60 μs in France, in Brazil this value remains below 0.20 μs in 90% of the cases.

Table III: RMS-DS for the measured Brazilian in-home PLC channels.

	RMS-DS (μs)	
	Band A	Band B
Maximum	0.493	0.465
Minimum	0.039	0.029
Mean	0.148	0.133
Std	0.064	0.064
50^{th} percentile	0.140	0.127
90^{th} percentile	0.227	0.204

A. Statistical Modeling

An exploratory analysis was performed in the ACA and RMS-DS datasets for the Brazilian in-home PLC channels. In this case, a statistical modeling for the ACA and RMS-DS features was performed considering several symmetric and asymmetric distributions, chosen according to the general behavior observed in each dataset. The considered symmetric distributions are the Logistic and the Normal, while the asymmetric ones are the Exponential, Gamma, Inverse Gaussian, Loglogistic, Lognormal, Nakagami, Rayleigh, Rician, Skew-normal, t-Student and Weibull.

The suitability of the fitting between the dataset and the statistical distributions was evaluated in terms of the log-likelihood function, as given in Eq. (5). Also, three information criteria (AIC, BIC and EDC) were employed to evaluate the proposed fitting, penalizing the number of parameters in each distribution to avoid data overfitting. These criteria have the general form of [10]

$$-2l(\hat{\boldsymbol{\theta}}) + Kc_n, \tag{7}$$

where K is the number of model parameters and c_n is the penalty term for the associated criterion as listed in Table IV, where n is the length of the data set. In contrary to the log-likelihood function, a lower information criterion value, as given in Eq. (7), corresponds to a better fitting between the dataset and the considered distribution.

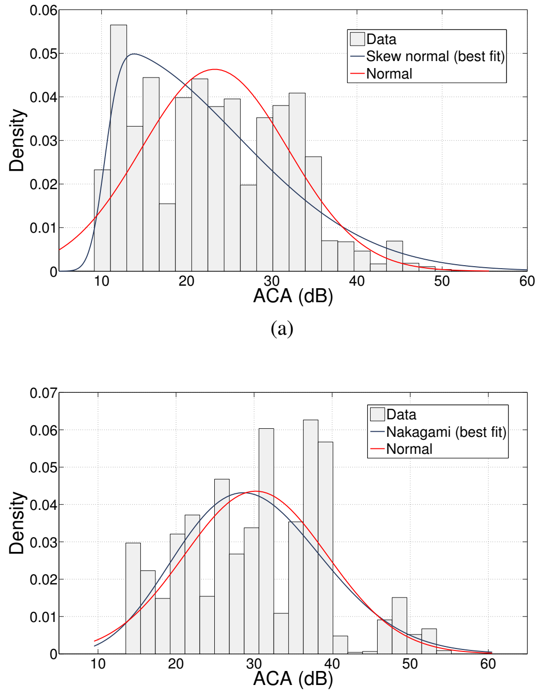
Table IV: Penalty term c_n of different information-based model evaluation criteria: AIC, BIC and EDC.

Criterium	c_n
AIC	2
BIC	$\log(n)$
EDC	$0.2\sqrt{n}$

The modeling parameters for the best fitted and the normal distributions for the ACA for both frequency bandwidths considered in this work are summarized in Table V.

As depicted in Fig. 1a, the attained results revealed that in Band A the ACA for Brazilian in-home PLC channels is better fitted, according to all evaluation criteria, by the Skew-normal distribution, even though this distribution has three parameters. For the same frequency range, the US PLC channels presented a normal ACA distribution, as detailed in [1]. For the sake of comparison, the best normal distribution for the Band A ACA in Brazil is also shown in Fig. 1a.

For Band B, the best ACA fit corresponds to the Nakagami distribution, as seen in Fig. 1b, although the normal distribution, also depicted in this figure, yielded quite similar values for the log-likelihood function and all adopted information criteria, as given in Table V. In [4], the best normal fit for the ACA related to PLC channels in Italy has the mean and standard deviation, (μ, σ), equal to $(35.412, 10.521)$ dB, against $(30.211, 9.158)$ dB for the best normal fit for the ACA Brazilian case. These results reinforce the fact that PLC channels in Italy suffer from an additional 5 dB attenuation in comparison to their Brazilian counterparts.



(a)

(b)

Figure 1: ACA distribution fitting: (a) Band A; (b) Band B.

With respect to the RMS-DS modeling, the histogram depicted in Fig. 2 shows a clear asymmetrical nature. In fact, the analysis of the RMS-DS for Brazilian in-home PLC channels revealed that this PLC channel feature is better modeled by a Gamma distribution, for both considered bandwidths. This result is different of those reported in [1], for US in-home PLC channels and for the frequency Band A, and in [4], for in-home PLC channels in Italy for frequency Band B, in which the RMS-DS was considered log-normally distributed. On the other hand, the results achieved by fitting the RMS-DS of Brazilian in-home PLC channels with the log-normal distribution are not so distant to those obtained for the Gamma distribution, as depicted in Fig. 2 and detailed in Table VI.

Table V: MLE results for the ACA statistical distribution fitting (SE are the estimated standard errors).

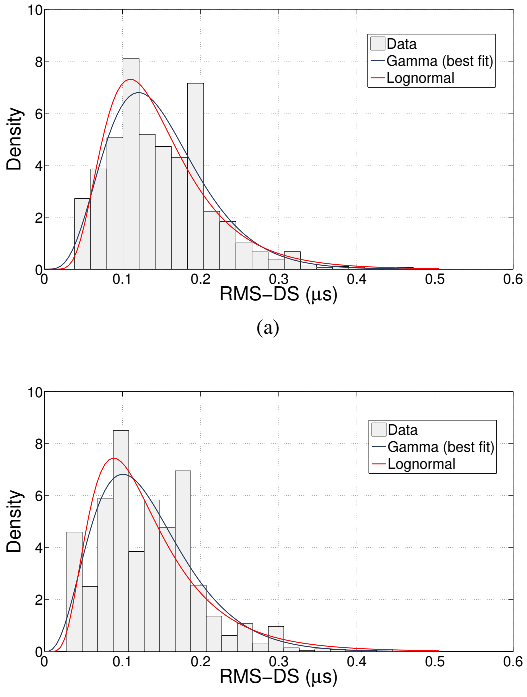
ACA for Band A							
Distribution	Parameter	Estimate	SE	Log-likelihood	AIC	BIC	EDC
Skew-normal	μ σ γ	22.7062 9.4122 0.9625	0.0124 0.0081 0.0015	$-2.3359\ 10^4$	$4.6726\ 10^4$	$4.6731\ 10^4$	$4.6768\ 10^4$
Normal	μ σ	23.2804 8.6094	0.0112 0.0056	$-2.3606\ 10^4$	$4.7215\ 10^4$	$4.7219\ 10^4$	$4.7244\ 10^4$

ACA for Band B							
Distribution	Parameter	Estimate	SE	Log-likelihood	AIC	BIC	EDC
Nakagami	μ ω	2.8123 996.5721	0.0021 53.4341	$-2.3962\ 10^4$	$4.7928\ 10^4$	$4.7931\ 10^4$	$4.7956\ 10^4$
Normal	μ σ	30.2112 9.1581	0.0127 0.0063	$-2.4014\ 10^4$	$4.8032\ 10^4$	$4.8035\ 10^4$	$4.8060\ 10^4$

Table VI: MLE results for the RMS-DS statistical distribution fitting (SE are the estimated standard errors).

RMS-DS for Band A							
Distribution	Parameter	Estimate	SE	Log-likelihood	AIC	BIC	EDC
Gamma	a b	5.3806 0.0275	0.0083 0.0002	$9.2350\ 10^3$	$-1.8466\ 10^4$	$-1.8462\ 10^4$	$-1.8437\ 10^4$
Lognormal	μ σ	-2.1459 0.5263	$0.4191\ 10^{-4}$ $0.2096\ 10^{-4}$	$9.0471\ 10^3$	$-1.8090\ 10^4$	$-1.8087\ 10^4$	$-1.8062\ 10^4$

RMS-DS for Band B							
Distribution	Parameter	Estimate	SE	Log-likelihood	AIC	BIC	EDC
Gamma	a b	4.1178 0.0322	0.0048 0.0001	$9.2151\ 10^3$	$-1.8426\ 10^4$	$-1.8423\ 10^4$	$-1.8398\ 10^4$
Lognormal	μ σ	-2.0058 0.4485	$0.3044\ 10^{-4}$ $0.1522\ 10^{-4}$	$9.1779\ 10^3$	$-1.8352\ 10^4$	$-1.8348\ 10^4$	$-1.8323\ 10^4$



(a)

(b)

Figure 2: RMS-DS distribution fitting: (a) Band A; (b) Band B.

VI. CONCLUSION

This work presented statistical models for the average channel attenuation (ACA) and the root mean squared value for the delay spread of measured in-home PLC channels in Brazil. Two different frequency bands were considered (Band A: from 1.7 up to 30 MHz; Band B: from 1.7 up to 100 MHz), allowing direct comparisons with several works presented in the literature. The provided analyses revealed that for Bands A and B the ACA can be better modeled by the skew-normal and the Nakagami distributions, respectively. This result is different from those reported for in-home PLC channels in US and in Italy, where the ACA was considered normally distributed. The results for the RMS-DS revealed that such PLC channel feature can be better modeled by a Gamma distribution for both frequency bands. Although this result is different to those reported for in-home PLC channels in US and in Italy, where the RMS-DS is considered lognormally distributed, the evaluation metrics (log-likelihood function, AIC, BIC and EDC) for the Gamma and the lognormal distributions were quite similar. Future works will be performed in order to determine statistical models for other PLC channel features such as the coherence time and the coherence bandwidth.

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