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ON WLS-CHEBYSHEV FIR DIGITAL FILTERS*

PAULO S. R. DINIZ
Programa de Engenharia Elétrica/DEL-EE, COPPE/Federal University of Rio de Janeiro, PO Box 68504, Rio de Janeiro, RJ, 21945-970, Brazil

and

SERGIO L. NETTO
Programa de Engenharia Elétrica/DEL-EE, COPPE/Federal University of Rio de Janeiro, PO Box 68504, Rio de Janeiro, RJ, 21945-970, Brazil

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A new numerical approach for designing FIR digital filters is proposed. The method is able to compromise maximum stopband attenuation and minimum stopband energy requirements. The approach is based on the weighted-least-squares (WLS) method using, at each iteration, a different weight function, which is made constant within a given frequency interval. In that manner, digital filters with partially equiripple and partially WLS-like stopbands are efficiently obtained. Generality of the method makes it suitable for the design of linear- and arbitrary-phase FIR filters.
FIR Filters
WLS and Chebyshev Criteria

1. Introduction

The design of finite-duration impulse response digital filters is dominated in the literature by the Chebyshev and the weighted-least-squares (WLS) approaches. The Chebyshev scheme minimizes the maximum absolute value of an error function between the prototype's transfer function and a given ideal solution. For that reason, Chebyshev filters are also said to satisfy a minimax criterion. The WLS approach, which minimizes the mean-squared-value of the same error function as the minimax approach, is characterized by a very simple implementation. Its basic problem, however, is the resulting Gibbs oscillations which correspond to large errors near discontinuities of the desired response.

The universal availability of minimax computer routines, including platforms based on high-level programming languages, has motivated its spread use in many problems where it is not the most appropriate solution. In fact, some applications that use narrow-band filters, like frequency division multiplexing for communications, do require both the maximum stopband attenuation and the total stopband energy to be considered simultaneously. For these cases, Adams[1−3] has shown that both the Chebyshev and WLS approaches are unsuitable as they completely dis-

*This paper was recommended by

regard one of these two measurements in their design procedure. For that matter, we propose a new approach for designing peak-constrained digital filters with low stopband energy.

The organization of this paper is as follows: In the next section, the problem of designing linear-phase nonrecursive digital filters is presented. In Section 3, the classical optimization methods for solving such approximation problem are described. These methods include the Chebyshev and the WLS approaches, the Lawson algorithm[4], and the so-called Lim-Lee-Chen-Yang (LLCY) algorithm[5]. The last two are seen as methods that implement the Chebyshev approach through a series of WLS designs. In Section 4, a new method is given based on a simple modification of the Lawson and LLCY algorithms, resulting in an excellent compromise of all good properties associated to both the Chebyshev and WLS methods. Section 5 includes extensions of the proposed method to the design of FIR filters with arbitrary phase.

2. Problem Formulation

Consider a nonrecursive filter of length N described by the transfer function

$$\tilde{H}(z) = \sum_{n=0}^{N-1} h(nT) z^{-n} \tag{1}$$

and assume that $\omega_s = 2\pi$, such that $T = 1$. The frequency response of such filter is then given by

$$\tilde{H}(e^{j\omega}) = \sum_{n=0}^{N-1} h(n) e^{-j\omega n} = e^{j\hat{\theta}(\omega)} \hat{H}(\omega) \tag{2}$$

where $\hat{\theta}(\omega)$ and $\hat{H}(\omega)$ are the phase and magnitude responses of $\tilde{H}(e^{j\omega})$ respectively defined as

$$\hat{\theta}(\omega) = \tan^{-1} \left\{ \frac{\text{Im}[\tilde{H}(e^{j\omega})]}{\text{Re}[\tilde{H}(e^{j\omega})]} \right\} \tag{3a}$$

$$\hat{H}(\omega) = \left| \tilde{H}(e^{j\omega}) \right| \tag{3b}$$

We concentrate our efforts here on linear-phase FIR filters, which are characterized by a phase response $\hat{\theta}(\omega)$ linear on ω, due to their practical importance. Assume then that N is odd[1], $h(n)$ is symmetrical[a]. The frequency response of such filter thus becomes

$$\tilde{H}(e^{j\omega}) = e^{-j\frac{(N-1)}{2}\omega} \sum_{n=0}^{c} a_n \cos(n\omega) \tag{4}$$

[a] Other cases of N even and/or $h(n)$ antisymmetrical can be dealt with in a very similar way[6] and are not further discussed in this paper. The arbitrary-phase FIR case is treated later as an extension of the linear-phase FIR problem.

with $c = \frac{(N-1)}{2}$, $a_0 = h(c)$, and $a_n = 2h(c-n)$, for $n = 1, \ldots, c$. Clearly, such response corresponds to a linear-phase filter.

If $e^{-jc\omega}H(\omega)$ is the desired frequency response and $W(\omega)$ is a strictly positive weighting function, consider the weighted error function $E(\omega)$ defined in the frequency domain as

$$E(\omega) = W(\omega)[H(\omega) - \hat{H}(\omega)] \tag{5}$$

The approximation problem for linear-phase nonrecursive digital filters resumes to the minimization of some objective function of $E(\omega)$ in such way that $|E(\omega)| \leq \delta$, and then

$$|H(\omega) - \hat{H}(\omega)| \leq \frac{\delta}{W(\omega)} \tag{6}$$

Evaluating the weighted error function on a dense frequency grid with $0 \leq \omega_i \leq \pi$, for $i = 1, \ldots, MN$, a good discrete approximation of $E(\omega)$ can be obtained. For practical purposes, for a filter of length N, using $8 \leq M \leq 16$ is recommended. Points associated to the transition band are disregarded, and the remaining frequencies should be linearly redistributed in the passband and stopband to include their corresponding edges. Thus, the following vector equation results

$$\mathbf{e} = \mathbf{W}\left(\mathbf{h} - \mathbf{U}\mathbf{a}\right) \tag{7}$$

where

$$\mathbf{e} = \left[E(\omega_1)\ E(\omega_2)\ \ldots\ E(\omega_{\bar{M}N})\right]^T \tag{8a}$$

$$\mathbf{W} = \operatorname{diag}\left[W(\omega_1)\ W(\omega_2)\ \ldots\ W(\omega_{\bar{M}N})\right] \tag{8b}$$

$$\mathbf{h} = \left[H(\omega_1)\ H(\omega_2)\ \ldots\ H(\omega_{\bar{M}N})\right]^T \tag{8c}$$

$$\mathbf{U} = \begin{bmatrix} 1 & \cos(\omega_1) & \cos(2\omega_1) & \ldots & \cos(c\omega_1) \\ 1 & \cos(\omega_2) & \cos(2\omega_2) & \ldots & \cos(c\omega_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \cos(\omega_{\bar{M}N}) & \cos(2\omega_{\bar{M}N}) & \ldots & \cos(c\omega_{\bar{M}N}) \end{bmatrix} \tag{8d}$$

$$\mathbf{a} = \left[a_0\ a_1\ \ldots\ a_c\right]^T \tag{8e}$$

with $\bar{M} < M$, as the original frequencies in the transition band were discarded.

An ideal lowpass filter is represented in Fig. 1, where δ_p is the passband maximum ripple, δ_s is the stopband minimum attenuation, and ω_p and ω_s are the passband and stopband edges, respectively.

Based on these values, define

$$DB_p = 20\log_{10}\left(\frac{1+\delta_p}{1-\delta_p}\right)\ \text{dB} \tag{9a}$$

$$DB_s = 20\log_{10}(\delta_s)\ \text{dB} \tag{9b}$$

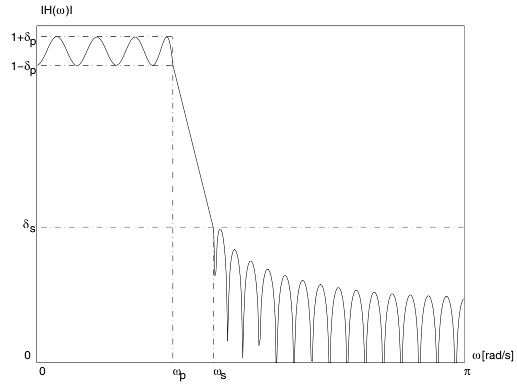


Fig. 1. Typical lowpass filter specifications.

The design of a lowpass digital filter as specified in Fig. 1, using either the minimax method or the WLS approach, is achieved making the ideal response and weight functions respectively equal to[6]

$$H(\omega) = \begin{cases} 1, & 0 \le \omega \le \omega_p \\ 0, & \omega_s \le \omega \le \pi \end{cases} \tag{10}$$

and

$$W(\omega) = \begin{cases} 1, & 0 \le \omega \le \omega_p \\ \delta_p/\delta_s, & \omega_s \le \omega \le \pi \end{cases} \tag{11}$$

3. Classical Optimization Approaches

3.1. *Chebyshev Method*

Chebyshev filter design consists of the minimization over the set of filter coefficients of the maximum absolute value of $E(\omega)$, i.e.,

$$\| E(\omega) \|_\infty \quad = \min_{\mathbf{a}} \max_{0 \le \omega \le \pi} [W(\omega)|H(\omega) - \hat{H}(\omega)|] \tag{12}$$

With the discrete set of frequencies, using Eq. (8), the Chebyshev method attempts to minimize

$$\| E(\omega) \|_\infty \quad \approx \min_{\mathbf{a}} \max_{0 \le \omega_i \le \pi} [\mathbf{W}|\mathbf{h} - \mathbf{U}\mathbf{a}|] \tag{13}$$

Referring to Fig. 1, the Chebyshev method effectively minimizes

$$DB_\delta = 20 \log_{10}(\delta) \text{ dB} \tag{14}$$

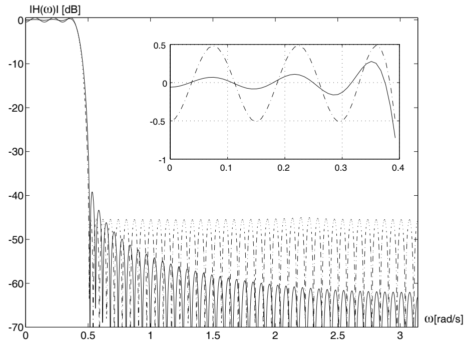


Fig. 2. Typical lowpass magnitude response (passband in detail) of Chebyshev- (dash-dotted curve) and WLS-based (solid curve) filters.

where $\delta = \max[\delta_p, \delta_s]$. An important feature of Chebyshev filters is their equiripple magnitude responses as seen in Fig. 2 (dash-dotted curve)[7].

3.2. *Weighted Least-Squares Method*

The weighted least-squares (WLS) approach minimizes

$$\| E(\omega) \|_2^2 \ = \int_0^{\pi} |E(\omega)|^2 \mathrm{d}\omega = \int_0^{\pi} W^2(\omega) |H(\omega) - \hat{H}(\omega)|^2 \mathrm{d}\omega \tag{15}$$

For the discrete set of frequencies, using Eq. (8), this objective function is estimated by

$$\| E(\omega) \|_2^2 \ \approx \mathbf{e}^T \mathbf{e} \tag{16}$$

the minimization of which is achieved with

$$\mathbf{a}^* = \left(\mathbf{U}^T \mathbf{W}^2 \mathbf{U} \right)^{-1} \mathbf{U}^T \mathbf{W}^2 \mathbf{h} \tag{17}$$

Referring to Fig. 1, the WLS objective is to maximize the passband-to-stopband ratio (PSR) of energies

$$PSR = 10 \log_{10} \left(\frac{\int_0^{\omega_p} |\hat{H}(\omega)|^2 \mathrm{d}\omega}{\int_{\omega_s}^{\pi} |\hat{H}(\omega)|^2 \mathrm{d}\omega 2} \right) \ \mathrm{dB} \tag{18}$$

A typical lowpass digital filter designed with the WLS method is depicted in Fig. 2 (solid curve), where the large ripples near the band edges are easily identified.

3.3. *Lawson and Lim-Lee-Chen-Yang Algorithms*

In 1961, Lawson derived a scheme that performs Chebyshev approximation as a limit of a special sequence of weighted least-p (L_p) approximations with p fixed.

The particular case with $p = 2$ thus relates the Chebyshev approximation to the WLS method. The L_2 Lawson algorithm is implemented by a series of WLS approximations using a varying weight matrix $\mathbf{W}_k$, the elements of which are calculated by [4]

$$W_{k+1}^2(\omega) = W_k^2(\omega) B_k(\omega) \tag{19}$$

where

$$B_k(\omega) = |E_k(\omega)| \tag{20}$$

Convergence of the Lawson algorithm is slow, as usually 10 to 15 WLS designs are required in practice to approximate the Chebyshev solution. An efficiently accelerated version of the Lawson algorithm was presented in[5]. This approach is hereby referred to as the Lim-Lee-Chen-Yang (LLCY) algorithm and is characterized by the weight matrix $\mathbf{W}_k$ recurrently updated by

$$W_{k+1}^2(\omega) = W_k^2(\omega) Be_k(\omega) \tag{21}$$

where $Be_k(\omega)$ is the envelope function of $B_k(\omega)$ formed by a set of piecewise linear segments that start and end at consecutive extremals of $B_k(\omega)$. Band edges are considered extremal frequencies, and edges from different bands are not connected. In that manner, labeling the extremal frequencies at a particular iteration k as ω_J^*, for $J = 1, 2, \ldots$, the envelope function is formed as[5]

$$Be_k(\omega) = \frac{(\omega - \omega_J^*) B_k(\omega_{J+1}^*) + (\omega_{J+1}^* - \omega) B_k(\omega_J^*)}{(\omega_{J+1}^* - \omega_J^*)}; \; \omega_J^* \leq \omega \leq \omega_{J+1}^* \tag{22}$$

Fig. 3 depicts typical cases of the absolute value of the error function (dash-dotted curve) used by the Lawson algorithm to update the weighting function, and its corresponding envelope (solid curve) used by the LLCY algorithm.

4. A New Approach

Comparing the adjustments used by the Lawson and LLCY algorithms, described in (19)–(22), and seen in Fig. 3, with the piecewise-constant weight function used by the WLS method, one can devise a very simple approach for designing digital filters that compromise both minimax and WLS constraints. The new approach consists of a modification on the weight-function updating procedure in such way that it becomes constant after a particular extremal of the stopband of $B_k(\omega)$, i.e.,

$$W_{k+1}^2(\omega) = W_k^2(\omega) \beta_k(\omega) \tag{23}$$

where, for the modified-Lawson algorithm, $\beta_k(\omega)$ is defined as

$$\beta_k(\omega) \equiv \tilde{B}_k(\omega) = \begin{cases} B_k(\omega), & 0 \leq \omega \leq \omega_J^* \\ B_k(\omega_J^*), & \omega_J^* < \omega \leq \pi \end{cases} \tag{24}$$

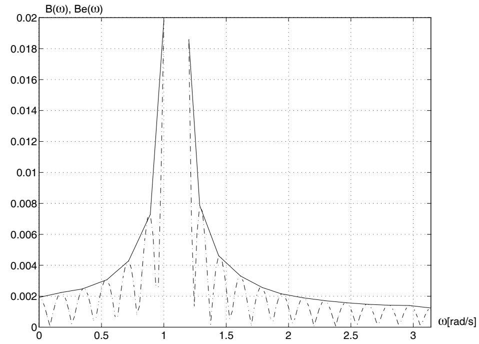


Fig. 3. Typical absolute error function $B(\omega)$ (dash-dotted line) and corresponding envelope $B_e(\omega)$ (solid curve).

and for the modified-LLCY algorithm, $\beta_k(\omega)$ is given by

$$\beta_k(\omega) \equiv \tilde{B}e_k(\omega) = \begin{cases} Be_k(\omega), & 0 \leq \omega \leq \omega_J^* \\ Be_k(\omega_J^*), & \omega_J^* < \omega \leq \pi \end{cases} \tag{25}$$

where ω_J^* is the J-th extreme value of the stopband of $B(\omega) = |E(\omega)|$. The passband values of $B(\omega)$ and $Be(\omega)$ are left unchanged in Eqs. (24) and (25) to preserve the equiripple property of the minimax method. The parameter J is the single design parameter for the proposed scheme. Choosing $J = 1$, makes the new scheme similar to an equiripple-passband WLS design. On the other hand, choosing J as large as possible, i.e., making $\omega_J^* = \pi$, turns the proposed scheme into the Lawson or the LLCY algorithms.

An example of the new approach being applied to the functions seen in Fig. 3 is depicted in Fig. 4, where ω_J^* was chosen as the fifth extremal in the filter's stopband.

The computational complexity of WLS-based algorithms, like the algorithms here described, is of the order of N^3, where N is the length of the filter. This burden, however, can be greatly reduced by taking advantage of the Toeplitz-plus-Hankel internal structure of the matrix $(\mathbf{U}^T\mathbf{W}^2\mathbf{U})$ in (17), as mentioned in[8,9], and by using an efficient grid scheme to minimize the number of frequency values, as described in[10,11]. These simplifications make the computational complexity of WLS-based algorithms comparable to the one for the minimax approach. The WLS-based methods, however, do have the additional advantage of being easily coded into computer routines.

Example 1: To illustrate the utilization of the proposed approach, a lowpass filter satisfying[1] $N = 95$, $DB_p = 1$ dB, $w_p = 2\pi 0.0625$ rad/s, and $w_s = 2\pi 0.0804$ rad/s was designed for all possible values of $1 \leq J \leq 42$. The resulting plot for DB_s and PSR, defined in (9b) and (18), respectively, is seen in Fig. 5(a). From this figure,

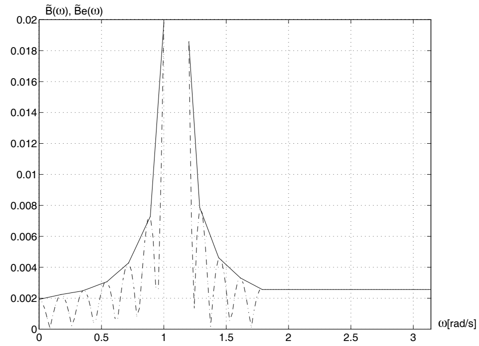


Fig. 4. New approach applied to the functions in Fig 3: Modified-Lawson algorithm $\tilde{B}(\omega)$ (dash-dotted curve) and Modified-LLCY algorithm $\tilde{B}e(\omega)$ (solid curve). Both curves coincide for $\omega \geq \omega_5^*$.

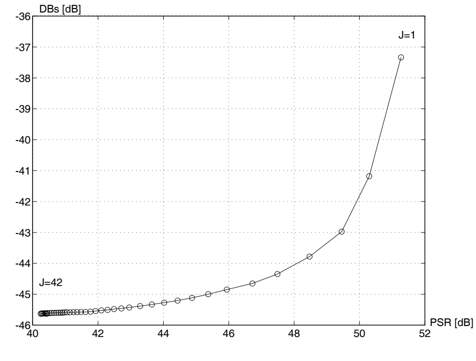
one can easily verify the poor results obtained with the minimax ($J = 42$) and WLS-like methods ($J = 1$), when considering both figures of merit simultaneously. For the sake of comparison, the same filter was designed using the Dolph-Chebyshev window as given in[6], resulting in $DBs = -26.66$ dB and $PSR = 21.26$ dB.

The magnitude response of the particular case when $J = 10$ is seen in Fig. 5(b), from where one can notice the partially WLS-like and partially equiripple (up to its tenth extremal) stopband and the equiripple passband. These characteristics are typical of the filters designed with the new approach.

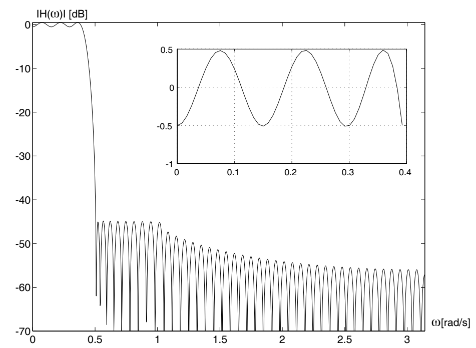
Both modifications given in (24) and (25) were used for the design described above yielding similar results. For all practical purposes, the modified-LLCY algorithm was able, in general, to reach a good neighborhood (within 1% of the specifications) of its final solution in about 5 iterations less than the modified-Lawson algorithm. However, when forcing the final results to be precise up to the third decimal place of $DB_p = 1.000$ (i.e., within 0.1%), the difference between the convergence speeds became quite considerable, as the modified-LLCY algorithm needed about 15 iterations and the modified-Lawson algorithm required the order of 200 iterations.

Example 2: In this example, a lowpass filter described by $DBp = 1$ dB, $DBs = -45.64$ dB, $\omega_p = 2\pi 0.0625$ rad/s, and $\omega_s = 2\pi 0.0804$ rad/s was designed with the new approach. The filter length necessary to satisfy the prescribed specifications was then measured for all possible values of $1 \leq J \leq 42$. The resulting plot of $N \times J$ is depicted in Fig. 6.

Notice that a small value of J may result in a very small increase of the filter length compared to its minimum value. In this case, for example, the minimum filter order was $N = 95$, obtained when $J = 42$ (which is equivalent to the minimax approach), and for $J = 5$, we had $N = 99$. In that manner, an excellent compromise



(a)



(b)

Fig. 5. Ex. 1 (a) $DBs \times PSR$ as functions of J. (b) Lowpass magnitude response (passband in detail) when $J = 10$ using the new approach.

between DBs and PSR can be achieved with a small value of J, as demonstrated in the previous example, with a very small increase of the filter length, as verified here.

Example 3: In this example, a bandpass filter was designed using the new approach for all distinct values of $1 \leq J \leq 23$. The filter specifications were $DBp = 1$ dB, $w_{s1} = (\pi/2 - 0.1)$ rad/s, $w_{p1} = (\pi/2 - 0.05)$ rad/s, $w_{p2} = (\pi/2 + 0.05)$ rad/s, $w_{s2} = (\pi/2 + 0.1)$ rad/s, and a filter length of $N = 95$. The plot of $DBs \times PSR$ for this design is shown in Fig. 7(a), from which one can verify the poor performances presented by the minimax ($J = 23$) and the WLS-like ($J = 1$) algorithms, if both figures of merit are considered at the same time. The magnitude response of the particular case when $J = 10$ is seen in Fig. 7(b).

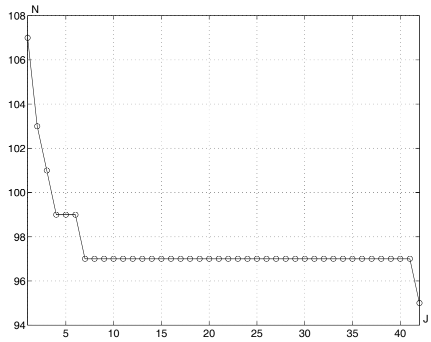


Fig. 6. Ex. 2 - Filter length N as a function of the extremal counter J.

Both the Lawson and the LLCY variations were used yielding very similar results. Once again, however, the LLCY version outperformed the modified-Lawson with respect to convergence speed.

5. Arbitrary-Phase FIR Filters

The problem of designing arbitrary-phase nonrecursive filters is described by generalizing Eq. (7) to the complex domain, i.e.,

$$\tilde{\mathbf{e}} = \mathbf{W}(\tilde{\mathbf{h}} - \tilde{\mathbf{U}}\tilde{\mathbf{a}}) \tag{26}$$

where

$$\tilde{\mathbf{h}} = \left[\tilde{H}(e^{j\omega_1})\ \tilde{H}(e^{j\omega_2})\ \ldots\ \tilde{H}(e^{j\omega_{\bar{M}N}})\right]^T \tag{27a}$$

$$\tilde{\mathbf{U}} = \begin{bmatrix} 1 & e^{-j\omega_1} & e^{-2j\omega_1} & \ldots & e^{-(N-1)j\omega_1} \\ 1 & e^{-j\omega_2} & e^{-2j\omega_2} & \ldots & e^{-(N-1)j\omega_2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & e^{-j\omega_{\bar{M}N}} & e^{-2j\omega_{\bar{M}N}} & \ldots & e^{-(N-1)j\omega_{\bar{M}N}} \end{bmatrix} \tag{27b}$$

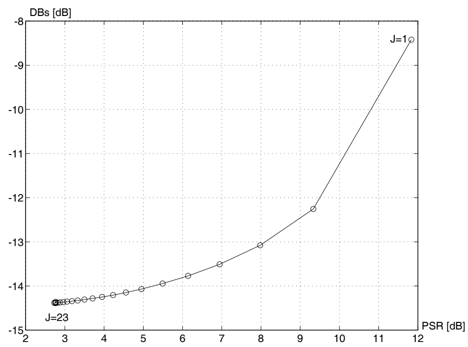
$$\tilde{\mathbf{a}} = [a_0\ a_1\ \ldots\ a_c]^T \tag{27c}$$

with $\tilde{H}(e^{j\omega})$ as defined in (2). The WLS minimization of $\tilde{\mathbf{e}}^H\tilde{\mathbf{e}}$ is then achieved by

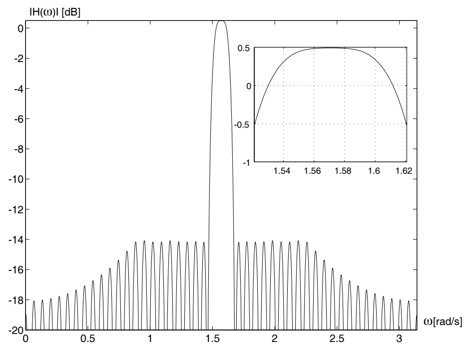
$$\tilde{\mathbf{a}} = \left[\mathrm{Re}(\tilde{\mathbf{U}}^H)\ \mathbf{W}^2\ \mathrm{Re}(\tilde{\mathbf{U}}) + \mathrm{Im}(\tilde{\mathbf{U}}^H)\ \mathbf{W}^2\ \mathrm{Im}(\tilde{\mathbf{U}})\right]^{-1} \cdot \left[\mathrm{Re}(\tilde{\mathbf{U}}^H)\ \mathbf{W}^2\ \mathrm{Re}(\tilde{\mathbf{h}}) + \mathrm{Im}(\tilde{\mathbf{U}}^H)\ \mathbf{W}^2\ \mathrm{Im}(\tilde{\mathbf{h}})\right] \tag{28}$$

In this case, however, the updating procedure for the weighting function can be based on the auxiliary error function defined as

$$\hat{E}(\omega) = W(\omega)\left[|\tilde{H}(e^{j\omega})| - |\hat{H}(e^{j\omega})|\right] \tag{29}$$



(a)



(b)

Fig. 7. Ex. 3 (a) $DBs \times PSR$ as functions of J. (b) Bandpass magnitude response (passband in detail) when $J = 10$ using the new approach.

instead of the original error function in (5).

Example 4: In this example, the proposed method is used to design a nonrecursive filter, the desired response of which is specified as[12]

$$\tilde{H}(e^{j\omega}) = \begin{cases} e^{-j\tau_s\omega}, & 0 \leq \omega \leq \omega_p \\ 0, & \omega_s \leq \omega \leq \pi \end{cases} \tag{30}$$

with $\omega_p = 2\pi 0.06$ rad/s, $\omega_s = 2\pi 0.12$ rad/s, and $\tau_s = 12$. In addition, we have $N = 31$ and $\delta_p/\delta_s = 10$. The trade-off between DBs and PSR for this example is depicted in Fig. 8. The case when $J = 14$, which is equivalent to the equiripple solution, resulted in $\delta_p = 0.03538$ and $\delta_s = 0.003536$. These values are considerably better than the results mentioned in[12]. Using $J = 3$, we obtain $\delta_p = 0.04427$ and $\delta_s = 0.004423$, which are comparable to the results in[12], with an additional 3.2 dB for the PSR, as seen in Fig. 8.

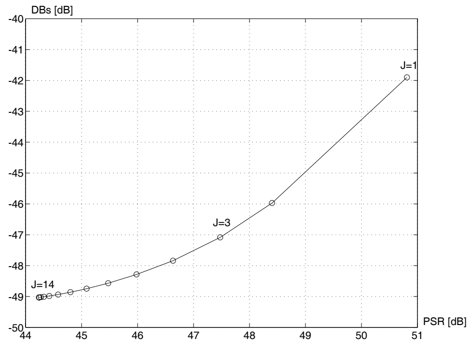


Fig. 8. Ex. 4 - $DBs \times PSR$ as functions of J.

The resulting magnitude and group-delay responses are shown in Fig. 9(a) and Fig. 9(b), with $J = 3$ (solid curve) and $J = 14$ (dash-dotted curve), respectively. Notice how close the delay is in both cases to the specified value $\tau_s = 12$ in the filter's passband.

6. Conclusion

A simple method for designing FIR digital filters was presented. The method is based on a modification of the so-called Lawson and Lim-Lee-Chen-Yang algorithms, forcing the corresponding weight function to be constant during a given frequency interval. The easy implementation of the method along with the resulting combination of the Chebyshev and WLS qualities indicate that the new approach represents a very efficient form of compromising the stopband's peak and energy constraints. In addition, the method has shown to be extremely general in the sense that it is suitable for approximating arbitrary-phase FIR filters.

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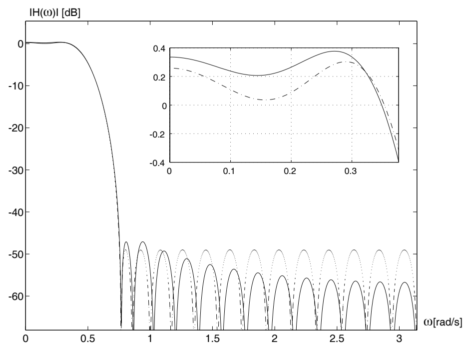
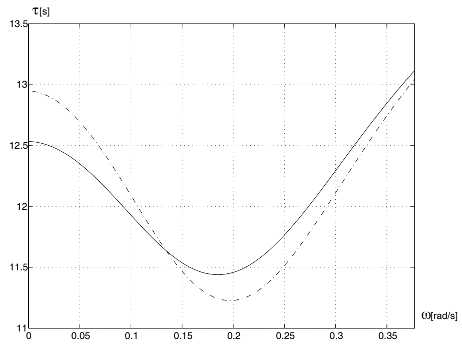


(a)



(b)

Fig. 9. Ex. 4 Frequency response using the new approach for $J = 3$ (solid curve) and $J = 14$ (dash-dotted curve): (a) Magnitude (passband in detail); (b) Passband group-delay.

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