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OPTIMIZATION OF FRM FILTERS USING THE WLS-CHEBYSHEV APPROACH*

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Abstract. This paper presents efficient methods for designing linear-phase finite impulse response filters by combining the frequency-response masking (FRM) approach and the weighted least-squares (WLS)-Chebyshev method. We first use the WLS-Chebyshev method to design quasi-equiripple FRM filters, achieving better performances with respect to the passband ripple or the stopband attenuation, when compared with the standard FRM design. Then, by exploiting the concept of critical bands, introduced in this paper, we present a method for designing modified FRM filters with a further reduction in the computational complexity. This is achieved by properly relaxing the specifications for the FRM base and masking filters and using the ability of the WLS-Chebyshev method to trade off the minimum attenuation and the total energy in the filter's stopband. Computational savings are in the order of 10%–15% of the original number of coefficients of the standard FRM design (using the concept of "don't care" bands for the masking filters).
Key words: frequences-response masking, digital filter design, FIR digital filters, weight-least squares method.

1. Introduction

The frequency-response masking (FRM) approach is a very efficient alternative for designing linear-phase finite impulse response (FIR) digital filters with wide passbands and sharp transition bands. Such a method, by allowing an increase in the filter group delay, enables one to reduce the filter complexity (number of multipliers and adders required per output sample) when compared with the standard design methods [6]. In fact, it has been verified, by means of an example, that with the FRM approach without the concept of "don't care" bands for the masking filters, the complexity is reduced to about 48% of the complexity required by the

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standard minimax approach. When using the concept of "don't care" bands, the resulting computational complexity is about 35% of the standard minimax design.

The weighted least-squares (WLS)-Chebyshev method is a design approach that attempts to minimize the stopband's total energy and maximum ripple simultaneously [1]. As presented in [3], the WLS-Chebyshev method can be implemented by a series of simple WLS designs, whose weight matrix is modified at each iteration according to some error function determined in the previous iteration.

In this paper, the FRM and WLS-Chebyshev methods are combined to design quasi-equiripple FRM filters, improving their performance with respect to the passband ripple and/or the stopband attenuation. This is achieved by properly performing an iterative optimization procedure for the base, positive masking, and negative masking filters, minimizing the margin for the error obtained by the previously designed filters. Advantages of the proposed scheme over previous FRM modifications [2]–[5] include the optimization of the FRM filter through a series of quadratic subproblems that are straightforward to perform, presenting suitable convergence characteristics such as a low complexity, robustness, and speed.

It is then illustrated that just turning the FRM filter into a quasi-equiripple solution is not very effective in achieving an overall order reduction, because it does not take into account the different sensitivities to coefficient variation in each band when designing the FRM subfilters. Using this motivation, the concept of critical bands is introduced. These are two bands, one at each side of the transition band of the overall filter, where there is a poor ripple cancellation between the two FRM branches, and the resulting ripple becomes significant. We then use the WLS-Chebyshev algorithm to design the base and masking filters, closely constraining the peaks at these critical bands. The resulting further reduction in the computational complexity of the resulting filter is approximately 15% of that of the standard FRM approach.

The organization of this paper is as follows. In Sections 2 and 3, we describe the main concepts behind the FRM and WLS-Chebyshev methods, respectively. In Section 4, we use the WLS-Chebyshev method for iteratively designing quasi-equiripple FRM filters. In Section 5, we define the critical bands and use this concept for designing a modified FRM filter with a reduced computational complexity. In Section 6, we combine the algorithms presented in Sections 4 and 5 to generate a design procedure for quasi-equiripple and computationally efficient FRM filters. Examples are provided throughout the paper to illustrate the effectiveness of the proposed techniques.

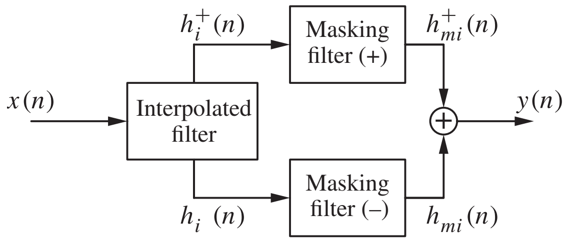


Figure 1. Realization of a reduced-order FIR filter with the FRM approach.

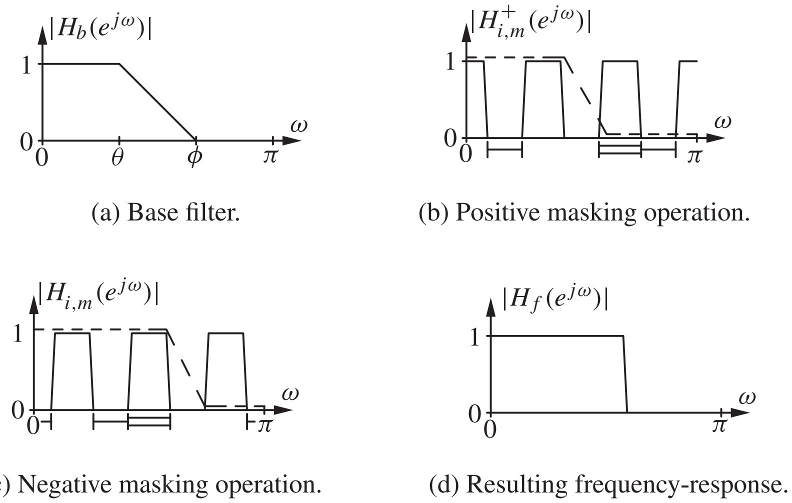


Figure 2. FRM approach, showing the "don't care" bands (single line) and the critical bands (double lines below the frequency axis).

2. FRM approach

The basic block diagram for the FRM approach is shown in Figure 1. In this scheme, the interpolated base filter presents a repetitive frequency spectrum, which is processed by the positive masking filter, of order N_{m_+}, in the upper branch of the FRM realization. Similarly, a complementary version of this repetitive frequency response is cascaded with the negative masking filter, of order N_{m_-}, in the lower branch of the realization. In this procedure, both masking filters keep some of the spectrum repetitions, which are then added together to compose the desired overall frequency response. The magnitude responses of the filter composing this sequence of operations are depicted in Figure 2, where the resulting filter presents a very sharp transition band.

The cutoff frequencies θ and ϕ of the base filter of an even order N_b (see

Figure 2) depend on the interpolation factor L and on the desired bandedge frequencies ω_p and ω_r of the overall filter. The masking filters are simple FIR filters with bandedge frequencies that also depend on L and on the bands of the interpolated filter. The optimal value of L that minimizes the overall number of multiplications can be obtained by estimating the orders of all subfilters for various L and finding the best-case scenario as given in [8].

To determine the ripple specifications for each subfilter, we notice that in the noncritical bands the resulting ripple is approximately the ripple of the corresponding masking filter (depending on the frequency value) with a second-order error term associated to the base filter, due to the almost-perfect cancellation of the two FRM branches. Therefore, in these bands, the FRM subfilters can have approximately the same ripple (about 98%) as the overall FRM filter. In the critical bands, the ripple cancellation is poor, and the FRM subfilters should have about 50% of the ripple allowed to the complete FRM filter. A detailed discussion on the precise calculation of these ripples is provided in [6].

3. WLS-Chebyshev algorithm

In the WLS-Chebyshev approach, one is able to positively combine the large attenuation characteristic of the Chebyshev (minimax) method with the low stopband energy characteristic of the WLS approximation methods [1]. In fact, the WLS-Chebyshev design scheme yields a filter response with a partially equiripple and partially least-squares-like stopband response. In [3] a very simple method was proposed for designing WLS-Chebyshev filters.

If an Nth-order FIR filter has symmetric impulse response with N even, its amplitude response can be written as [4]

$$|\hat{H}(e^{j\omega})| = \left| \sum_{i=0}^{N/2} \hat{a}(i)\mathrm{trig}(\omega, i) \right|, \tag{1}$$

where $\hat{a}(i)$ are the filter coefficients and $\mathrm{trig}(\omega, i)$ denotes a proper trigonometric function [4]. Using a dense grid of frequency values ω_n for $n = 1, \ldots, F$, equation (1) can be expressed as

$$\hat{\mathbf{h}} = \hat{\mathbf{a}}^T \mathbf{U}, \tag{2}$$

where

$$\hat{\mathbf{h}} = [\, \hat{H}(e^{j\omega_1})\ \hat{H}(e^{j\omega_2})\ \ldots\ \hat{H}(e^{j\omega_F})]^T \tag{3}$$

$$\hat{\mathbf{a}} = [\, \hat{a}(0)\ \hat{a}(1)\ \ldots\ \hat{a}(N/2)]^T \tag{4}$$

$$\mathbf{U} = \begin{bmatrix} 1 & \mathrm{trig}(\omega_1) & \mathrm{trig}(2\omega_1) & \ldots & \mathrm{trig}(N\omega_1/2) \\ 1 & \mathrm{trig}(\omega_2) & \mathrm{trig}(2\omega_2) & \ldots & \mathrm{trig}(N\omega_2/2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \mathrm{trig}(\omega_F) & \mathrm{trig}(2\omega_F) & \ldots & \mathrm{trig}(N\omega_F/2) \end{bmatrix}. \tag{5}$$

The WLS solution in practice minimizes the objective function

$$\epsilon = \sum_{n} r(\omega_n) E^2(\omega_n), \tag{6}$$

where $r(\omega)$ is a nonnegative weighting function and $E(\omega)$ is the amplitude error with respect to the ideal response $H(e^{j\omega})$, that is,

$$E(\omega) = |H(e^{j\omega})| - |\hat{H}(e^{j\omega})|. \tag{7}$$

The WLS solution is then given by [4]

$$\hat{\mathbf{a}}^* = \left(\mathbf{U}^T \mathbf{R}^2 \mathbf{U}\right)^{-1} \mathbf{U}^T \mathbf{R}^2 \mathbf{h} \tag{8}$$

where $\mathbf{R}$ is a diagonal matrix with samples of the weighting function, and $\mathbf{h}$ is a vector with samples of the desired response, that is,

$$\mathbf{R} = \text{diag}[\ r(\omega_1)\ r(\omega_2)\ \ldots\ r(\omega_F)]^T \tag{9}$$

$$\mathbf{h} = [\ H(e^{j\omega_1})\ H(e^{j\omega_2})\ \ldots\ H(e^{j\omega_F})]^T. \tag{10}$$

We can use a series of WLS designs to achieve the Chebyshev (minimax) solution by using in equations (6)–(9) a variable weighting function at each iteration k, as given by [9]

$$r_{k+1}(\omega_n) = \beta_k(\omega_n) r_k(\omega_n), \tag{11}$$

where

$$\beta_k(\omega_n) = \frac{|E_k(\omega_n)|}{\sum_{m} r_k(\omega_m)|E_k(\omega_m)|}. \tag{12}$$

An example of such a function is depicted by the dashed curve in Figure 3a. An accelerated version proposed in [7] updates $r_k(\omega_n)$ with the envelope of $\beta_k(\omega_n)$ defined above. This envelope is determined by searching the peaks of $|E_k(\omega_n)|$ for every ω_n and using a piecewise function to join these extreme points, as illustrated by the solid curve in Figure 3a.

If the updating of the weighting function in equation (11) is made constant for a given frequency interval $\omega \in [\omega^*, \pi]$, where ω^* is the Jth stopband peak, the resulting frequency response becomes WLS-like within this band and equiripple in the remaining frequencies [3]. In such a case, the modified envelope function is as shown in Figure 3b, whereas the typical magnitude response of the corresponding WLS-Chebyshev filter is as depicted in Figure 4.

The main advantage of the WLS-Chebyshev scheme is the total control over the resulting response provided by the method. Such control is easily and completely determined by the weighting function, which specifies the frequencies where one accepts or desires more or less gain. Such an ability is very important for modifying the FRM design to generate quasi-equiripple filters, as will be described in Section 4.

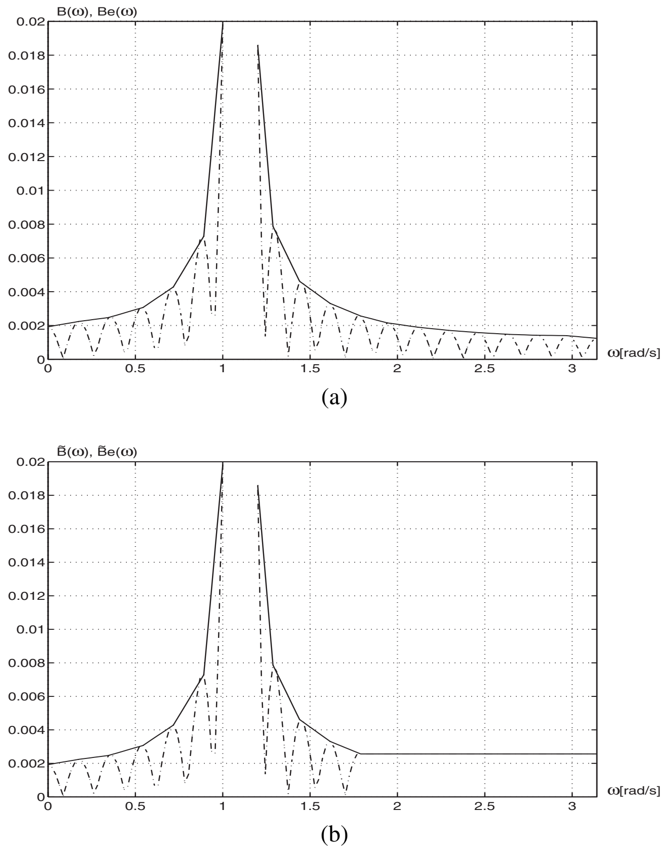


Figure 3. Weighting function $\beta_k(\omega_n)$: (a) Equiripple method, original (dashed curve) and accelerated (solid curve) schemes; (b) WLS-Chebyshev method, classic (dashed curve) and accelerated (solid curve) schemes.

4. Quasi-equiripple FRM filter

In the standard FRM design, the base and masking filters are designed to have equiripple responses. This can be achieved with the WLS-Chebyshev method by forcing $\omega_J = \pi$ in each basic filter design. To obtain a quasi-equiripple FRM filter, we then use the error function between this initial filter design and the desired response (see equation (7)), to adjust the weight function of the WLS-Chebyshev method, as given by equation (12), to redesign the FRM base and masking filters. The central idea here is to aim for an equiripple overall FRM filter, instead of equiripple FRM subfilters, as occurs in the original FRM design. The overall procedure can be made recursive until a predetermined number of

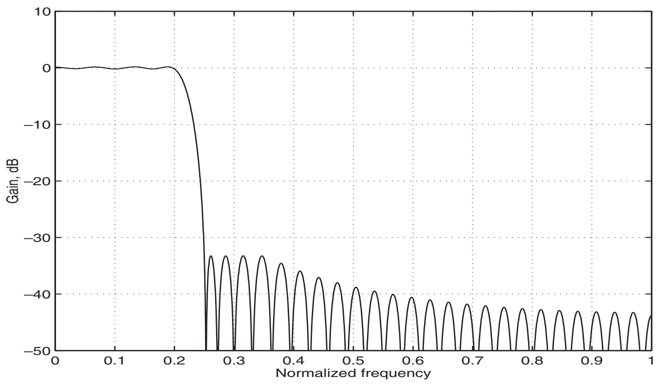
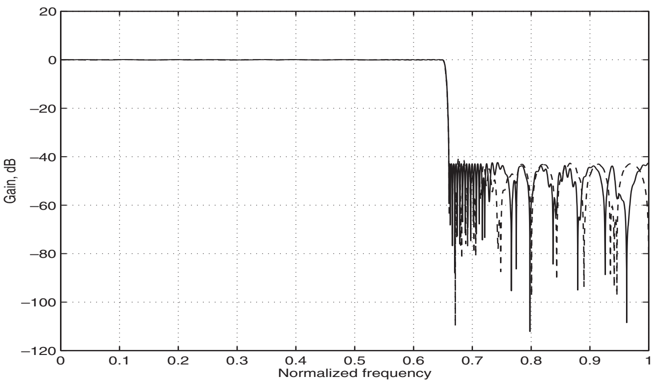


Figure 4. Typical magnitude response of a WLS-Chebyshev filter with $J = 5$, illustrating the partially equiripple and partially WLS-like stopband.

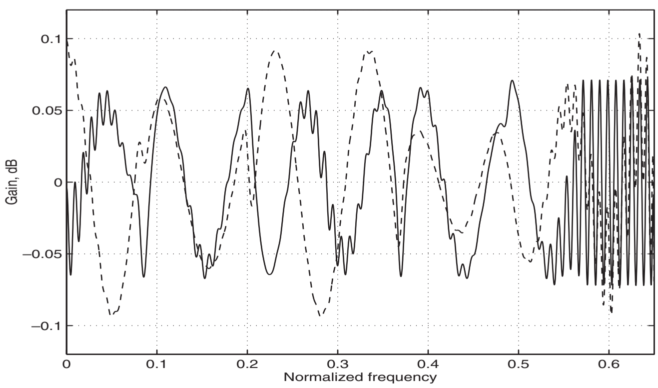
Table 1. Algorithm 1: Quasi-equiripple FRM algorithm using the WLS-Chebyshev scheme

Step 1.	Perform standard FRM design and set $aux = 0$;
Step 2.	Measure the error between the desired and the obtained responses using equation (7);
Step 3.	If the error is acceptable, then output the filter design and stop;
Step 4.	If $aux < 3$, then set $aux = aux + 1$; Else, set $aux = 1$;
Step 5.	If $aux = 1$, then `object=base`; If $aux = 2$, then `object=positive mask`; If $aux = 3$, then `object=negative mask`;
Step 6.	Adjust the weighting function of the `object` filter using equation (11), and redesign it with the WLS-Chebyshev scheme using equation (8);
Step 7.	Go to *Step 2*.

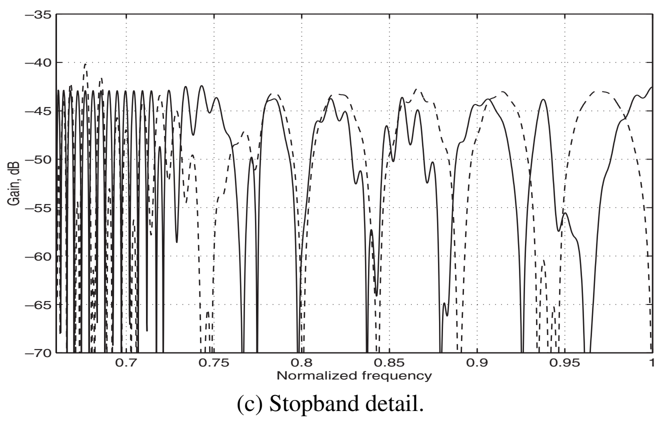
iterations has been performed or some filter characteristic remains unchanged in consecutive iterations. In the proposed scheme, after the design of each FRM basic filter, the error function is re-evaluated, and the weight function for the subsequent basic filter is adjusted. The detailed algorithm for the proposed quasi-equiripple design combining the FRM and WLS-Chebyshev methods is described in Table 1 and illustrated in Example 1. In Step 6 of Algorithm 1, to determine the coefficients of the `object` filter at hand, one must consider the coefficients of the other filters fixed, as given by the previous iteration of the algorithm. Once the `object` filter is optimized with the WLS-Chebyshev algorithm, it then becomes fixed, and a different FRM subfilter is selected as the `object` filter. Such a loop continues until convergence is achieved.



(a) Complete magnitude responses.



(b) Passband detail.



(c) Stopband detail.

Figure 5. Magnitude responses of standard (dashed line) and quasi-equiripple (solid line) FRM filters in Example 1.

Table 2. Filter characteristics using several FRM designs in Example 1

	Filter 1	Filter 2	Filter 3
A_p (dB)	0.1978	0.1435	0.1861
A_r (dB)	40.16	42.39	40.39
N_b	64	64	60
N_{m_+}	37	37	37
N_{m_-}	27	27	25
L	7	7	7
FRM order	485	485	457
Number of coefficients	66	66	63

Example 1. Consider the design of a lowpass filter with passband edge $\omega_p = 0.65\pi$, stopband edge $\omega_r = 0.66\pi$, maximum passband ripple $A_p = 0.2$ dB, and minimum stopband attenuation $A_r = 40$ dB. The standard minimax FIR filter requires an order $N = 381$, whereas the standard FRM filter (using the concept of "don't care" bands for the masking filters) satisfies the specifications with the characteristics of Filter 1 in Table 2, requiring a total of 66 distinct coefficients.

Using the WLS-Chebyshev algorithm to optimize the FRM filter, as described above, we obtain Filter 2, which is fully characterized in Table 2. The magnitude responses of Filter 1 (dashed line) and Filter 2 (solid line) are depicted in Figure 5, where one can clearly visualize the improvements in the passband ripple and the stopband attenuation achieved when using the quasi-equiripple design.

In a heuristic attempt to reduce the order of the FRM filter, to exploit the improvements resulting from the quasi-equiripple design, very little was achieved, as illustrated by the setup Filter 3 in Table 2. In such an experiment, several combinations of order reductions between the base and masking filters were attempted, yielding the best-case scenario with respect to the overall number of distinct coefficients given in Filter 3. As one can see, the resulting filter still satisfies all desired specifications, while providing only a mild reduction in the number of distinct coefficients, when compared to the previous two designs.

5. Efficient FRM filter

In this section, we consider the fact that in the FRM design there are two critical bands where the ripples in the two FRM branches have poor cancellations. These bands are the last passband of the interpolated base filter (or its complementary filter) within the FRM passband and the first passband of the complementary interpolated base filter (or of the interpolated base filter) within the FRM stopband. These critical bands are represented by the double lines under the frequency axis in Figure 2. An illustration of the concept of critical bands is seen in Figure 5. In such a figure, the dashed line, which represents the magnitude response of the standard FRM filter, presents peaks at the very end of the passband and at the very beginning of the stopband.

Based on the concept of critical bands, we then develop a new design technique to improve the FRM design using the WLS-Chebyshev scheme. The new approach starts from the basic FRM design. We first locate the repetition of the base filter spectrum which is responsible for the sharp transition of the filter. These frequencies are given by [6]

$$\omega_1 = m\frac{\pi}{L}; \quad \omega_2 = (m+1)\frac{\pi}{L}, \tag{13}$$

where m is the largest integer such that ω_2 is immediately below the largest cutoff frequency ω_s of the masking filters. These two frequencies, ω_1 and ω_2, are the centers of the first and second critical bands, respectively. Once these frequencies are found, we can take into consideration the effect of the masking filter responses over the base filter response, and estimate the resulting error as given in equation (7) by considering

$$\begin{aligned}|H(e^{j\omega})| &= |H_m^+(e^{j\omega})H_i^+(e^{j\omega}) + H_m^-(e^{j\omega})H_i^-(e^{j\omega})| \\ &= |H_m^+(e^{j\omega})H_i^+(e^{j\omega}) + H_m^-(e^{j\omega})[1-H_i^+(e^{j\omega})]|\end{aligned} \tag{14}$$

over the interval $\omega \in [\omega_1, \omega_2]$. As we are interested in optimizing the base filter, we can map the frequency responses of the masking filters back to the frequency interval $\omega \in [0, \pi]$, yielding

$$|H(e^{j\omega})| = |H_m^+(e^{j\omega'})H_b(e^{j\omega}) + H_m^-(e^{j\omega'})[1 - H_b(e^{j\omega})]|, \tag{15}$$

where

$$\omega' = \omega_1 + (\omega_2 - \omega_1)\frac{\omega}{\pi} \tag{16}$$

if the positive masking filter has cutoff frequencies below the negative masking filter, or

$$\omega' = \omega_2 - (\omega_2 - \omega_1)\frac{\omega}{\pi} \tag{17}$$

if the positive masking filter has cutoff frequencies above the negative masking filter cutoff. This definition of ω' means that depending on which of the two branches is responsible for the last part of the passband, one needs to do a direct or inverse frequency mapping, according to equation (16) or (17), respectively. The last step is to determine the peak-constrained frequencies. For this project, we use the first bandstop peak ('sidelobe') of the masking filter. In the frequencies above this peak, it is assumed that the least-squares part of the base filter will cancel the other peaks of the masking filters. Thus, in each iteration, we seek for the first bandstop peak to determine where the envelope function is kept constant. Once the peak-constrained frequencies are known, the optimization algorithm can be applied to design the base filter. The masking filters are redesigned as described in Section 4. The efficient FRM algorithm is summarized in Table 3. The main difference between Algorithm 2 and Algorithm 1 is that the latter is iterative, with

Table 3. Algorithm 2: Modified FRM algorithm using the WLS-Chebyshev scheme based on the critical band concept

Step 1.	Perform standard FRM design;
Step 2.	Determine critical peak $\omega^\star$ in critical bands;
Step 3.	Redesign the base filter with the WLS-Chebyshev scheme using $\omega_J = \omega^\star$ at critical bands;
Step 4.	Measure the error between the desired and the obtained responses using equation (7);
Step 5.	Adjust the weighting functions using equation (11), and redesign the masking filters using the WLS-Chebyshev scheme to obtain a quasi-equiripple FRM filter.

Table 4. Number of distinct coefficients for several frequency specifications, with $A_p =$ 0.2 dB and $A_r = 40$ dB, using the standard and modified FRM algorithms

Specifications			Standard FRM	Efficient FRM
ω_p	ω_s	L	No. of coefficients	No. of coefficients
0.178π	0.180π	14	141	123
0.240π	0.245π	10	91	83
0.32π	0.33π	8	67	60
0.65π	0.66π	7	66	58

all FRM subfilters being redesigned one at a time until convergence of the FRM overall filter is achieved, whereas in Algorithm 2 each FRM subfilter is redesigned only once, aiming at an equiripple FRM filter.

Example 2. In Table 4, we see the design results for several frequency specifications and the corresponding interpolation factor L that minimizes the number of distinct coefficients. In practice, the optimal value of L can be different for the standard and modified FRM algorithms. By using the same value of L in both algorithms, however, it is easier to compare the results, because the subfilters will keep the same frequency specifications for both algorithms. Notice that in all cases the modified FRM using the proposed WLS-Chebyshev algorithm results in filters requiring considerably fewer multipliers than those obtained with the standard FRM algorithm.

Example 3. Consider the specifications of Example 1. Analyzing the magnitude response within the stopband's critical band in Figure 6, we notice that the most critical peak is the fifth one, if we initialize $J = 1$ at $\omega = \omega_r$. Therefore, by specifying $J = 5$ equiripple peaks in the WLS-Chebyshev design for the base filter, we are able to constrain the overall response in the critical band. This simplifies the specifications for the masking filters, which are redesigned as in Section 4, aiming at an equiripple FRM filter. The resulting filter magnitude response using such an algorithm is shown in Figure 7 (dashed line), along with

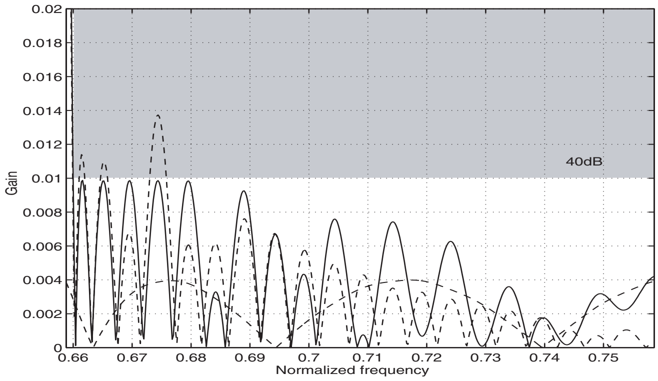


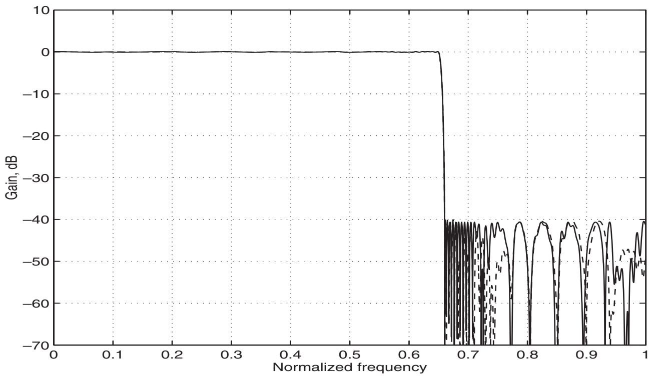
Figure 6. Critical-band detail of magnitude responses of the FRM filter (continuous line) and FRM base and masking filters (dashed lines).

the passband and stopband details. The characteristics of the modified FRM filter referred to as Filter 4 are included in Table 5.

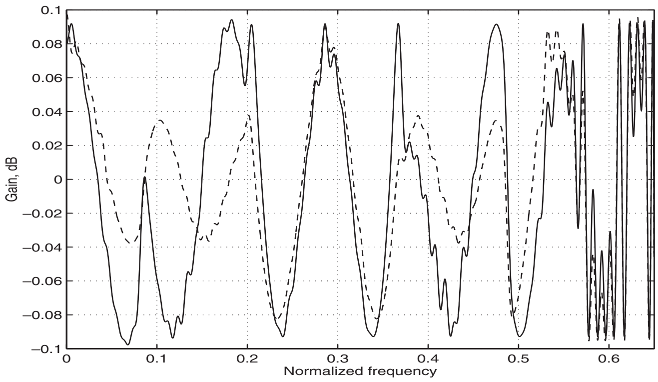
6. Quasi-equiripple efficient FRM filter

Comparing the two algorithms, we notice that whereas Algorithm 1, given in Section 4, is iterative (the FRM subfilters are repeatedly designed until a quasi-equiripple response is achieved), Algorithm 2, given in Section 5, is not. However, the main difference between these two methods is that Algorithm 1 uses more zeros within the critical bands than Algorithm 2, in order to force the quasi-equiripple response (see the stopband details in Figures 5 and 7, respectively). This occurs because Algorithm 2 judiciously solves the approximation problem in the critical bands with the WLS-Chebyshev scheme. In this way, Algorithm 2 has more degrees of freedom to minimize the objective function in the remaining bands, thus improving the overall FRM frequency response. These two algorithms can then be merged to obtain a modified FRM design. The idea is to start with Algorithm 2 to generate an initial design for Algorithm 1. In this manner, Algorithm 2 is used to obtain a computationally efficient design, while Algorithm 1 is iteratively used to force a quasi-equiripple response on the resulting filter.

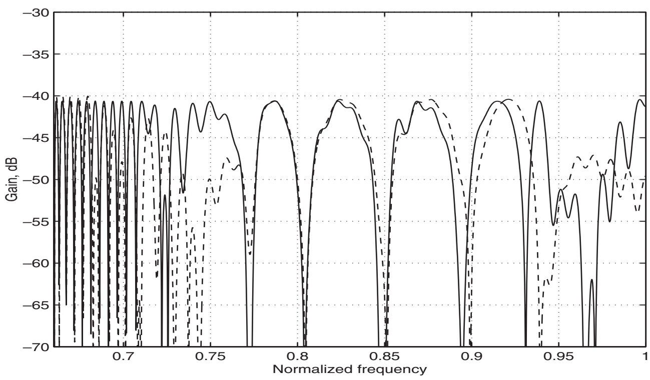
Example 4. Using the combination of Algorithm 2 and Algorithm 1 (in this order) to design the filter specified in Example 1, yields the setup Filter 5, whose characteristics are given in Table 5. From this table, we notice that the improvements of Filter 5 over Filter 4 are very small, illustrating that the design generated by Algorithm 2 has already presented a near-optimal response. The response of Filter 5 is seen in Figure 7 (solid line), where the quasi-equiripple nature is clearly observed.



(a) Complete magnitude responses.



(b) Passband detail.



(c) Stopband detail.

Figure 7. Magnitude response of efficient (dashed line) and quasi-equiripple efficient (solid line) FRM filters using the concept of critical bands in Examples 3 and 4.

Table 5. Filter characteristics using the concept of critical bands in Examples 3 and 4

	Filter 4	Filter 5
A_p (dB)	0.1960	0.1920
A_r (dB)	40.11	40.44
N_b	56	56
N_{m_+}	32	32
N_{m_-}	26	26
L	7	7
FRM order	424	424
Number of coefficients	58	58

7. Conclusions

Three modifications on the FRM design method for FIR filters were introduced. The proposed methods are based on the WLS-Chebyshev approach, which allows a greater flexibility to exploit the weighting function, given any desired response. In this manner, one can easily control the resulting magnitude response by increasing or decreasing the weight function in the desired frequency bands according to some predetermined error function. Using this approach, we can design FRM filters with relaxed sets of specifications, yielding computationally efficient FRM filters with quasi-equiripple responses. Examples indicate that the computational reduction is about 15% of the total computational complexity of the standard FRM filter (using the concept of "don't care" bands for the masking filters).

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