

EFFICIENT DESIGN OF NARROWBAND COSINE-MODULATED FILTER BANKS USING A TWO-STAGE FREQUENCY-RESPONSE MASKING APPROACH

SERGIO L. NETTO*, LUIZ C. R. de BARCELLOS†, and PAULO S. R. DINIZ‡

*Programa de Engenharia Elétrica-COPPE/EE,
 Federal University of Rio de Janeiro,
 P. O. Box 68504, Rio de Janeiro, RJ, 21945-970, Brazil*

* *sergioln@lps.ufrj.br*

† *barcellos@lps.ufrj.br*

‡ *diniz@lps.ufrj.br*

A new cosine-modulated filter bank (CMFB) structure is proposed based on the frequency-response masking (FRM) approach using masking filter decomposition. The resulting structure, the so-called FRM2-CMFB, presents reasonable computational complexity (number of arithmetic operations per output sample) and allows one to design filter banks with extremely large number of bands. The examples include the use of $M = 1024$ bands, where the standard minimax method cannot be employed. These examples indicate that the reduction in computational complexity can be as high as 60% of the original FRM-CMFB structure, which does not use masking filter decomposition.

Keywords:

1. Introduction

The frequency-response masking (FRM) approach is an efficient method for designing linear-phase FIR digital filters with general passbands and sharp transition bands. With such method, by allowing a small increase in the overall filter order, it is possible to reduce the number of coefficients to about 30% of the number required by a minimax FIR direct-form filter.¹

The cosine-modulated filter banks (CMFBs) are very popular in applications requiring large number of subbands due to their easy design (based solely on a single prototype filter) and computationally efficient implementation.^{2–6}

In this paper, we analyze the use of the FRM approach to design the prototype filter of a CMFB. The narrowband-CMFB case is considered where standard minimax method fails to converge and even the FRM-CMFB⁷ structure presents high computational complexity. A new structure is then introduced, the so-called FRM2-CMFB, where the original FRM masking filter is decomposed as a cascade of two filters, namely a second stage interpolated base filter followed by the corresponding masking filters. The result is the further reduction in the computational

complexity of the CMFB implementation, when compared to the FRM-CMFB, and the possibility of designing filter banks with large number of bands.

The organization of this paper is as follows: In Secs. 2 and 3, we describe the basic concepts behind the FRM and CMFB methods, respectively. In Sec. 4, the FRM-CMFB structure presented in Ref. 7 is revised. A narrowband CMFB design is then reviewed as a motivation to introduce the FRM2-CMFB structure in Sec. 5. The examples are included to illustrate the results achieved with the proposed method.

2. The FRM Method

In the FRM technique, a lowpass base filter is interpolated by a factor of L , $H_{b1}(z^L)$, what generates a repetitive frequency spectrum whose output signal is processed by the so-called positive masking filter, $G_1(z)$. Similarly, the complement of this repetitive frequency response, $H_{b2}(z^L)$, is cascaded by the negative masking filter, $G_2(z)$. In such scheme, both masking filters must keep the desired spectrum repetitions within the overall passband, while eliminating the undesired spectrum repetitions. The output of both masking filters are then added together to form the desired overall response. This entire procedure is represented by

$$H(z) = H_{b1}(z^L)G_1(z) + H_{b2}(z^L)G_2(z) \quad (1)$$

and is illustrated in Fig. 1, where one can clearly see the sharp transition nature of the resulting filter.

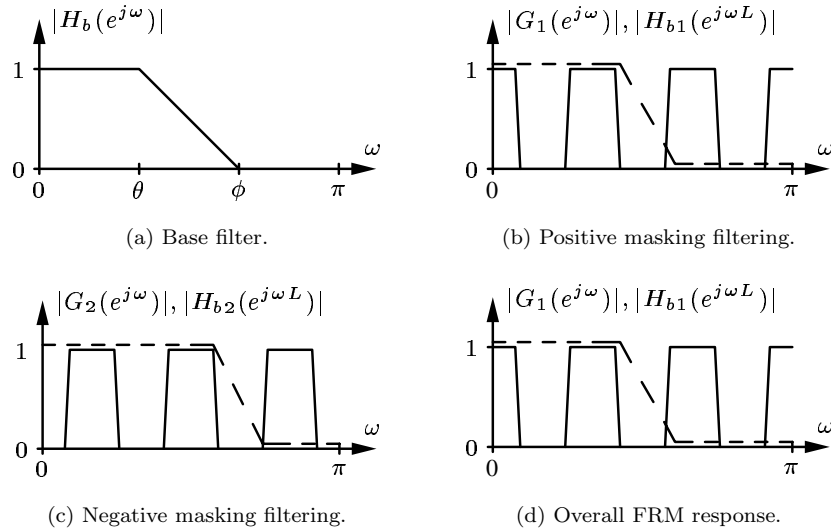


Fig. 1. FRM operation: The combination of the frequency responses in the direct and complementary branches generates the FRM filter with general passband and narrow transition band.

If the base filter has linear phase and an even order, N_b , its complementary version can be obtained by subtracting the output signal of $H_{b1}(z^L)$ from a delayed version of the input signal obtained at its central node, that is

$$H_{b2}(z^L) = z^{-N_b/2} - H_{b1}(z^L) = z^{-N_b/2} - \sum_{i=0}^{N_b} h_b(i)z^{-Li}, \quad (2)$$

where $h_b(n)$ is the impulse response of the base filter. Hence, under such conditions, no additional complexity is required to implement the complementary filter $H_{b2}(z^L)$ in the FRM scheme. If the passband is narrow enough when compared to the desired transition band, it is possible to eliminate the FRM complementary branch and further reducing the number of coefficients in the resulting filter.¹ Such case of CMFBs with large number of bands is addressed in this paper.

3. The CMFB Structure

CMFBs are a commonly used tool in signal processing applications.^{2,3,8,9} The main advantages of CMFBs include its simple design, as only one prototype filter is required, since the analysis and synthesis filter banks are obtained by modulating this filter with a proper set of cosine functions, and its computationally efficient implementation. The CMFB prototype filter is characterized by a 3-dB attenuation point and the stopband edge frequency given by

$$\omega_{3\text{dB}} \approx \frac{\pi}{2M}; \quad \omega_s = \frac{(1+\rho)\pi}{2M}, \quad (3)$$

where ρ is the so-called roll-off factor that controls the amount of overlapping between adjacent bands. If the prototype filter has order N_p and transfer function

$$H_p(z) = \sum_{n=0}^{N_p} h_p(n)z^{-n}, \quad (4)$$

then the impulse responses of the analysis and the synthesis filters are given by

$$h_m(n) = 2h_p(n) \cos \left[\frac{(2m+1)(n-N_p/2)\pi}{2M} + (-1)^m \frac{\pi}{4} \right], \quad (5)$$

$$f_m(n) = 2h_p(n) \cos \left[\frac{(2m+1)(n-N_p/2)\pi}{2M} - (-1)^m \frac{\pi}{4} \right], \quad (6)$$

respectively, for $m = 0, 1, \dots, (M-1)$, and $n = 0, 1, \dots, N_p$.

If the CMFB prototype filter has $(N_p + 1) = 2KM$ coefficients, then we can perform a polyphase decomposition on $H_p(z)$, with $2M$ components, as follows

$$\begin{aligned} H_p(z) &= \sum_{j=0}^{2M-1} \left[c_{m,j} z^{-j} \sum_{k=0}^{K-1} (-1)^k h_p(2kM+j) z^{-2kM} \right] \\ &= \sum_{j=0}^{2M-1} z^{-j} E_j(z^{2M}) \end{aligned} \quad (7)$$

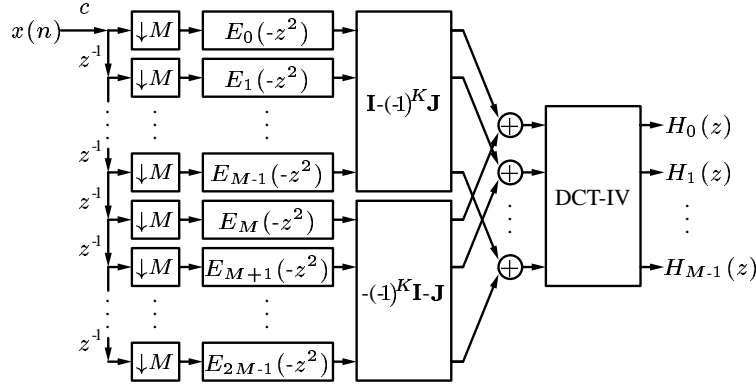
4 *S. L. Netto, L. C. R. de Barcellos & P. S. R. Diniz*

Fig. 2. Efficient CMFB structure.

with

$$E_j(z) = \sum_{k=0}^{K-1} h_p(2kM + j)z^{-k}, \quad (8)$$

for $j = 1, \dots, 2M$.Therefore, after some standard algebraic manipulations,² the analysis filters can then be written as^a

$$\begin{aligned} H_m(z) &= \sum_{j=0}^{2M-1} \left[c_{m,j} z^{-j} \sum_{k=0}^{K-1} (-1)^k h_p(2kM + j) z^{-2kM} \right] \\ &= \sum_{j=0}^{2M-1} c_{m,j} z^{-j} E_j(-z^{2M}), \end{aligned} \quad (9)$$

for $m = 0, 1, \dots, (M - 1)$. Based on such description, the analysis filter bank can be efficiently implemented as given in Fig. 2, where \mathbf{I} is the identity matrix, \mathbf{J} is the reverse identity matrix, and each element of the DCT-IV matrix is given by

$$[\mathbf{C}^{\text{IV}}]_{m,n} = \sqrt{\frac{2}{M}} \cos \left[\frac{2m+1}{2M} \left(n + \frac{1}{2} \right) \pi \right]. \quad (10)$$

4. The FRM-CMFB Structure

Let us consider the FRM-CMFB structure where the FRM approach is used to design the prototype filter of a CMFB. Assume also that we are mainly interested on the narrowband filter case, where the complementary branch is absent from the

^aA similar decomposition can be performed to the synthesis filters.

FRM structure.^b In such case, the transfer functions for the analysis filters are given by

$$H_m(z) = \sum_{n=0}^N c_{m,n} (h_{b_1}^I * g_1)(n) z^{-n}, \quad (11)$$

where the term $(h_{b_1}^I * g_1)(n)$ denotes the convolution between the interpolated base filter and the positive masking filter responses, and N is the overall order of the FRM filter. The key point in the FRM-CMFB structure is to find out how to obtain a computationally efficient polyphase decomposition of this convolution operation.

Assuming that $H_{b_1}(z)$ and $G_1(z)$ have orders N_b and N_m , respectively, and using the definition of convolution, Eq. (11) can be rewritten as

$$H_m(z) = \sum_{i=0}^{N_b} \left[h_b(i) z^{-Li} \sum_{n=0}^{N_m} c_{m,(n+Li)} g_1(n) z^{-n} \right]. \quad (12)$$

It can then be shown that if the interpolation factor can be written as⁷

$$L = 2K_a M + \frac{M}{K_b}, \quad (13)$$

where $K_a \geq 0$ and $K_b > 0$ are integer numbers, then the analysis filters can be expressed as

$$H_m(z) = \sum_{q=0}^{Q-1} \left[z^{-Lq} H'_{b_1q}(-z^{LQ}) \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K_b}q)} z^{-j} E'_j(-z^{2M}) \right], \quad (14)$$

where

$$H'_{b_1q}(z) = \sum_{k=0}^{K_c-1} (-1)^{K_a q} h_b(kQ + q) z^{-k}, \quad (15)$$

for $q = 0, 1, \dots, (Q-1)$, where $Q = 2K_b$ and $(N_b + 1) = QK_c$. Then, an efficient FRM-CMFB structure results, where the total number of coefficients is equal to

$$\mathcal{M} = N_b + N_m + 1 \quad (16)$$

without taking into account the lower branch of the FRM structure.

Example 1. In this example, we design a CMFB with $M = 1024$ channels, using a roll-off factor of $\rho = 0.1$, a maximum bandpass ripple of $A_p = 0.2$ dB, and a minimum stopband attenuation of $A_r = 50$ dB. The standard minimax design would require a prototype filter of order $N = 88\,865$, which is an impractical option.

^bThe general case is addressed, for instance, in Ref. 7.

6 *S. L. Netto, L. C. R. de Barcellos & P. S. R. Diniz*

For this design, by selecting $K_a = 0$ and $K_b = 4$, such that $L = 256$ (which corresponds to $Q = 8$ polyphase components for the base filter) we obtain a prototype filter described in Table 1. This design results on a total of $\mathcal{M} = 1147$ coefficients, corresponding to $\mathcal{M}_d = 574$ distinct coefficients. As we noted, the masking filter will have less than $2M$ coefficients, therefore several lines of the FRM-CMFB structure can be removed, simplifying the resulting filter even further.

Table 1. FRM-CMFB characteristics in Example 1.

| L | N_b | N_+ | N_- | \mathcal{M} | A_p | A_r |
|-----|-------|-------|-------|---------------|---------|-------|
| 256 | 344 | 801 | 0 | 1147 | 0.08 dB | 62 dB |

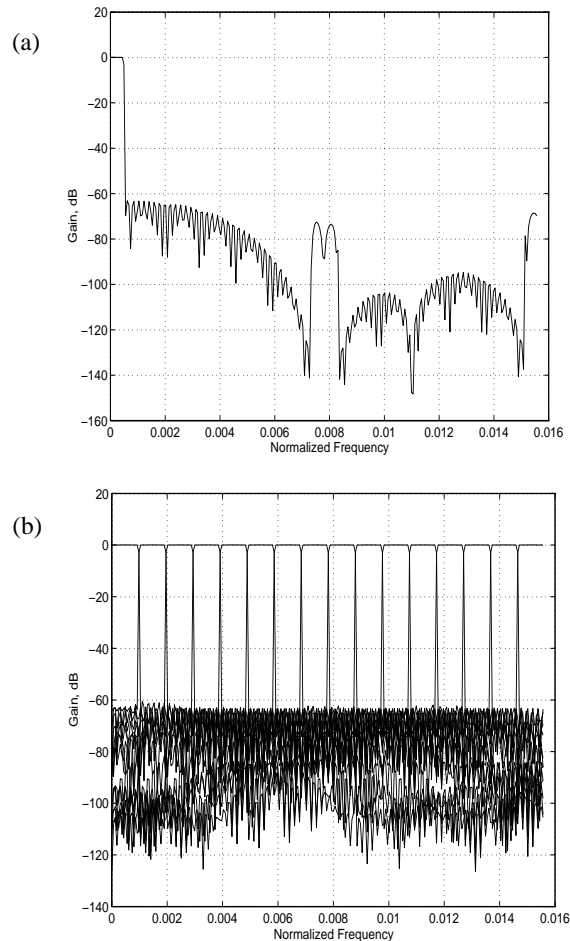


Fig. 3. Magnitude responses of Example 1: (a) Prototype filter; (b) Filter bank (bands 0 to 15).

The partial magnitude response for the prototype filter can be seen in Fig. 3(a), and the 16 first channels of the corresponding filter bank are depicted in Fig. 3(b).

5. The FRM2-CMFB Structure

As discussed above, the FRM method can be an efficient way to design CMFB prototype filters, particularly in cases where the number of bands tends to be very large, and the prototype-filter order reaches impractical levels. In fact, there are cases, as the one illustrated in Example 1 that even the FRM-CMFB tends to present reasonably high computational complexity. The structure can then be further simplified by designing the prototype filter with another stage of the FRM method to form the original masking filter. This technique is referred to as a two-stage FRM method, if only the upper branch is present in the original FRM filter (as in the case addressed in this paper), or masking filter factorization,¹⁰ if the two FRM branches are present in the original design.

Thus, by using the two-stage FRM method to design the CMFB prototype filter, the resulting structure, the so-called (FRM2-CMFB), is characterized by the block diagram depicted in Fig. 4. In the FRM2-CMFB structure, we can then write

$$H_m(z) = \sum_{n=0}^N c_{m,n} ((h_{b1}^I * h_b^{I'}) * g_1)(n) z^{-n}, \quad (17)$$

where $(h_{b1}^I * h_b^{I'})(n)$ denotes the convolution of the two interpolated base filters. Notice, however, that this convolution must present an overall interpolation factor that satisfies Eq. (13). For instance, if L is a multiple of L' , we can write

$$H_m(z) = \sum_{i=0}^{N_B} \left[(h_{b1}^{I_2} * h_b^{I'}) (i) z^{-L'i} \sum_{n=0}^{N_m} c_{m,(n+L'i)} g_1(n) z^{-n} \right], \quad (18)$$

where N_B is the order of the convolution $(h_{b1}^I * h_b^{I'})(n)$, and $h_{b1}^{I_2}$ represents the original base filter interpolated by a factor of L/L' . From Eq. (18), the values of $c_{m,(n+L'i)}$ depend only on L' , and therefore the two base filters together will not misalign the DCT-IV terms in the masking filter decomposition. We can then rewrite $H_m(z)$ as

$$H_m(z) = \sum_{q=0}^{Q'-1} \left[z^{-L'q} \bar{H}_{b1q}(-z^{L'Q'}) \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K'}q)} z^{-j} E'_j(-z^{2M}) \right], \quad (19)$$

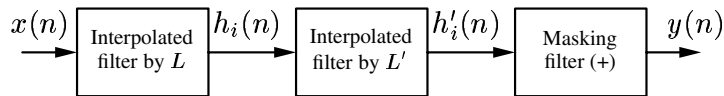


Fig. 4. CMFB prototype filter using the two-stage FRM method.

where Q' , L' , and K'_b are the respective counterparts of Q , L , and K_b , in the polyphase decomposition of the second base filter, $H'_b(z)$, and $\bar{H}_{b1q}(-z^{L'Q'})$ represents the z -transform of the convolutions between $h_{b1}^{I_2}$ and each polyphase component of $H'_b(z)$, the second base filter. It is possible, however, in the z domain to treat each of these convolutions as the cascade of two filters, turning Eq. (19) into

$$H_m(z) = H_{b1}(-z^L) \sum_{q=0}^{Q'-1} \left[z^{-L'q} H'_{bq}(-z^{L'Q'}) \sum_{j=0}^{2M-1} c_{m,(n+\frac{M}{K'_b}q)} z^{-j} E'_j(-z^{2M}) \right], \quad (20)$$

where $H'_{bq}(z)$ is the q th polyphase component of $H'_b(z)$, for $q = 0, 1, \dots, (Q' - 1)$. From this equation, we notice that by using the two-stage FRM method to design prototype filter for a CMFB, it is necessary to decompose the second-stage base filter by a factor of $Q' = 2M/L'$, and to introduce a slightly changed version of the interpolated base filter, $H_{b1}(-z^L)$, at the input of the FRM-CMFB. This leads to the so-called FRM2-CMFB structure depicted in Fig. 5.

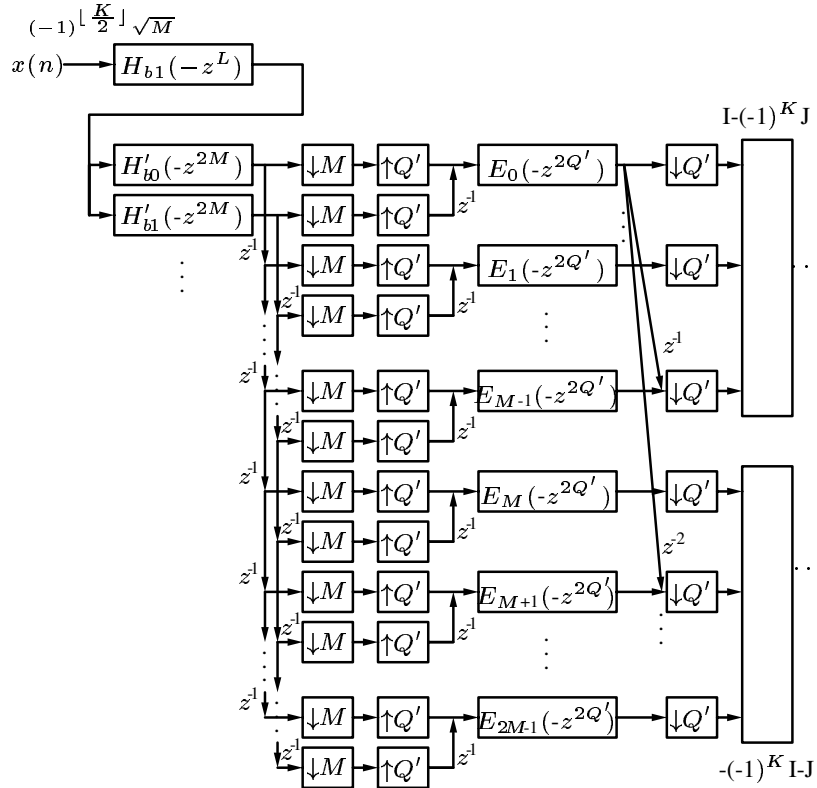


Fig. 5. Block diagram of the FRM2-CMFB using the two-stage FRM method to design the CMFB prototype filter (the DCT-IV block is not shown).

Table 2. FRM2-CMFB characteristics in Example 2.

| L' | N'_b | N'_+ | N'_- | \mathcal{M} | A_p | A_r |
|------|--------|--------|--------|---------------|---------|-------|
| 16 | 66 | 49 | 0 | 462 | 0.02 dB | 60 dB |

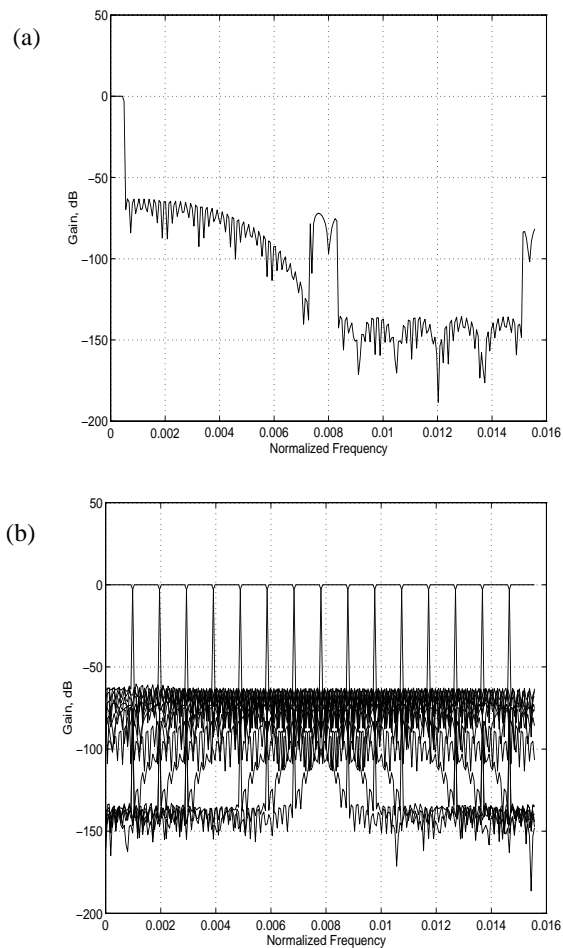


Fig. 6. Magnitude responses of Example 2: (a) Prototype filter; (b) Filter bank (bands 0 to 15).

Example 2. In this second example, we design the filter bank described in Example 1 using the FRM2-CMFB method. In this case, the original base filter remains unchanged while the masking filter is decomposed into two new filters. For the second-stage base filter the interpolation factor was chosen to be $L' = 16$, corresponding to $Q' = 128$, yielding the filter characteristics shown in Table 2 and the magnitude responses seen in Fig. 6. Thus, in this case, for the complete FRM2-

Table 3. FRM-CMFB characteristics in Example 3.

| L | N_b | N_+ | N_- | \mathcal{M} |
|------|-------|-------|-------|---------------|
| 1024 | 88 | 7613 | 0 | 7703 |

Table 4. FRM2-CMFB characteristics in Example 3.

| L' | N'_b | N'_+ | N'_- | \mathcal{M} | A_p | A_r |
|------|--------|--------|--------|---------------|----------|-------|
| 64 | 116 | 189 | 0 | 396 | 0.045 dB | 60 dB |

CMFB design only a total of $\mathcal{M} = 462$ coefficients is needed. Moreover, the structure may become even more compact, since the second-stage positive masking filter presents only 50 coefficients, while $Q' = 128$ polyphase components are used for the second-stage interpolated base filter.

Example 3. In this example, the same filter bank of Example 1 is designed with the FRM2-CMFB method, starting, however, from a FRM-CMFB with $L = M = 1024$, thus requiring only $Q = 2$ polyphase components for the base filter. In such case, the FRM-CMFB is characterized in Table 3, where we notice the extremely high order required by the original masking filter.

Using the FRM2-CMFB structure to reduce the computational complexity, with $L' = 64$, corresponding to $Q' = 32$ polyphase components for the second-stage base filter, we end up with the filter-bank characteristics shown in Table 4. From this table, one clearly notices that the total number of coefficients is drastically reduced to $\mathcal{M} = 396$, which is even smaller than the results achieved in Example 2. The magnitude responses for this FRM2-CMFB prototype filter and overall filter bank are seen in Fig. 7.

6. Conclusion

It was shown how the frequency-response masking (FRM) approach can be applied for designing the prototype filter in cosine-modulated filter banks (CMFBs). In narrowband cases (which arise when the number of filter bands become extremely large), it was verified that the standard minimax method requires a prohibitive filter order, and even the FRM-CMFB tends to present significant computational complexity. We then address this problem by introducing the so-called FRM2-CMFB structure which uses a two-stage FRM approach where the original FRM masking filter is decomposed into two filters, namely the second-stage interpolated base and masking filters. The design examples illustrated how the proposed FRM2-CMFB can significantly reduce the computational complexity of the FRM-CMFB structure in cases of narrowband CMFBs.

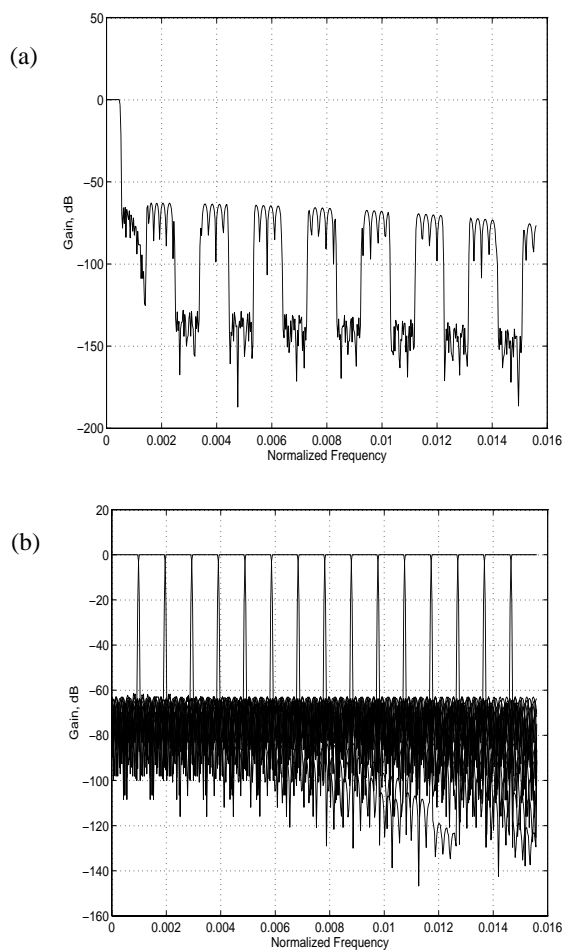


Fig. 7. Magnitude responses of Example 3: (a) Prototype filter; (b) Filter bank (bands 0 to 15).

References

1. Y. C. Lim, "Frequency-response masking approach for the synthesis of sharp linear phase digital filters", *IEEE Trans. Circuits Syst.* **CAS-33** (1986) 357–364.
2. P. P. Vaidyanathan, *Multirate Systems and Filter Banks*, Prentice-Hall, Englewood Cliffs, NJ, 1993.
3. T. Saramäki, "A generalized class of cosine-modulated filter banks", *Proc. TICSP Workshop Transforms and Filter Banks*, Tampere, Finland, June 1998, pp. 336–365.
4. A. Viholainen, T. Saramäki, and M. Renfors, "Cosine-modulated filter bank design for VDSL modems", *Proc. IEEE Int. Workshop Intelligent Signal Processing and Communication Systems*, Melbourne, Australia, November 1998, pp. 143–147.
5. J. Alhava and A. Viholainen, "Implementation of nearly perfect-reconstruction cosine-modulated filter banks", *Proc. FinSig*, Oulu, Finland, 1999, pp. 222–226.

12 *S. L. Netto, L. C. R. de Barcellos & P. S. R. Diniz*

6. A. Viholainen, T. Saramäki, and M. Renfors, “Nearly-perfect reconstruction cosine-modulated filter bank design for VDSL modems”, *Proc. IEEE Int. Conf. Electronics, Circuits Systems*, Paphos, Greece, September 1999, pp. 373–376.
7. P. S. R. Diniz, L. C. R. de Barcellos, and S. L. Netto, “Design of cosine-modulated filter bank prototype filters using the frequency-response masking approach”, *Proc. IEEE Int. Conf. Acoustics, Speech, and Signal Processing*, Vol. VI, Salt Lake City, UT, May 2001, pp. SPTM-P4.6 1–4.
8. H. S. Malvar, *Signal Processing with Lapped Transforms*, Artech House, Boston, MA, 1991.
9. P. S. R. Diniz, E. A. B. da Silva, and S. L. Netto, *Digital Signal Processing: System Analysis and Design*, Cambridge University Press, Cambridge, UK, 2002.
10. Y. C. Lim and Y. Lian, “Frequency-response masking approach for digital filter design: Complexity reduction via masking filter factorization”, *IEEE Trans. Circuits Syst. II, Analog and Digital Signal Processing* **41** (1994) 518–525.