Redundant Paraunitary FIR Transceivers for Single-Carrier Transmission Over Frequency Selective Channels With Colored Noise

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***Abstract*—Recently, the use of redundant memoryless single-carrier transmitters has been reported as an efficient choice to reduce distortions introduced by finite-impulse response (FIR) channels. In this work, redundant FIR transceivers are proposed to address not only channel frequency selectivity but additive colored noise, which strongly degrades the performance of memoryless transceivers. The transmitter is shown to be paraunitary, resulting in a simple receiver. The proposed system is optimized like a modulated filter bank. Channel shortening and post-combiner equalizers are used to improve system performance. Comparisons with recent proposed schemes are presented, illustrating the efficiency of the new structure for selective channels with colored noise.**

***Index Terms*—Communication systems, multiuser channels, paraunitary transceivers, single-carrier (SC) transmission, transmultiplexing.**

I. INTRODUCTION

RECENTLY, single-carrier (SC) digital modulation has regained attention due to its potential to overcome some drawbacks introduced by multicarrier modulation, such as peak-to-average ratio and subcarrier offset [1]. Moreover, for some frequency selective channels, the bit-error rate (BER) of an SC system can be considerably lower than the one obtained by a multicarrier modulation, specially if some subchannels face very strong attenuation due to channel imperfections [2]. The reason is that in multicarrier systems, the information of a given subchannel is usually concentrated in frequency and spread over time. If the channel is very poor in a given frequency range, the information of the respective subchannel may be lost.

In the dedicated literature, improved performances have been reported when using redundant memoryless SC transmitters (see, for example, [2]–[4]), but no work has reported the reduction of the BER for systems employing redundant finite-impulse response (FIR) transmitters. Since the performance of a communication system can be severely degraded by colored noise, this work proposes the design of redundant FIR transceivers based on lossless lattice structures to combat such possible impairments. Considering an ideal channel, the proposed system is shown to be a redundant perfect reconstruction (PR) transmultiplexer (TMUX). In order to reduce the channel influence on the BER, a synchronization delay (or channel shortening) is used, along with a post-combiner equalizer, which is trained in the domain of an orthogonal transform, such as the families of discrete cosine transforms (DCTs), discrete sine transforms (DSTs), and the discrete Fourier transform (DFT) [5].

This paper compares the performances of redundant FIR and zero-order transceivers for frequency selective channels, in presence of additive white Gaussian and colored noises. It is assumed that there is no *a priori* knowledge of the channel, but just a coarse estimation of its order, which represents a very practical case. A special optimization procedure is presented, which brings high selectivity to the receiver, enabling improved performance for the equalizer.

The paper is organized as follows. Section II introduces general FIR filter bank transceivers and notations. In Section III, the theory of the proposed paraunitary redundant filter bank is presented. In Section IV, both synchronization delay and post-combiner equalizer are described. The equalizer is designed using the minimum mean squared error (MMSE) criterion. Simulation results are presented in Section V to verify the efficiency of the proposed structure. The experiments are carried out for both real and complex channels. Conclusions are presented in Section VI.

II. GENERAL FIR FILTER BANK TRANSCEIVERS

The polyphase representations of causal analysis and synthesis filters in a transmultiplexer are given by

$$\begin{aligned} H_m(z) &= \sum_{n=0}^{N-1} E_{m,n}(z^N) z^{-(N-1-n)} \\ F_m(z) &= \sum_{n=0}^{N-1} R_{n,m}(z^N) z^{-n} \end{aligned} \tag{1}$$

for $m = 0, \dots, (M-1)$, respectively, where N is the downsampling/upsampling factor. In this manner, the causal analysis bank of an oversampled M-band filter bank can be defined as

$$\begin{aligned} \tilde{\boldsymbol{e}}(z) &= [H_0(z)\ H_1(z)\ \dots\ H_{M-1}(z)]^T \\ &= \boldsymbol{E}(z^N) z^{-N+1} \boldsymbol{d}^T(z^{-1}) \end{aligned} \tag{2}$$

where

$$\boldsymbol{E}(z^N) = \begin{bmatrix} E_{0,0}(z^N) & E_{0,1}(z^N) & \cdots & E_{0,N-1}(z^N) \\ E_{1,0}(z^N) & E_{1,1}(z^N) & \cdots & E_{1,N-1}(z^N) \\ \vdots & \vdots & \ddots & \vdots \\ E_{M-1,0}(z^N) & E_{M-1,1}(z^N) & \cdots & E_{M-1,N-1}(z^N) \end{bmatrix} \tag{3}$$

$$\boldsymbol{d}(z) = [1\ z^{-1}\ \cdots\ z^{-N+1}]^T. \tag{4}$$

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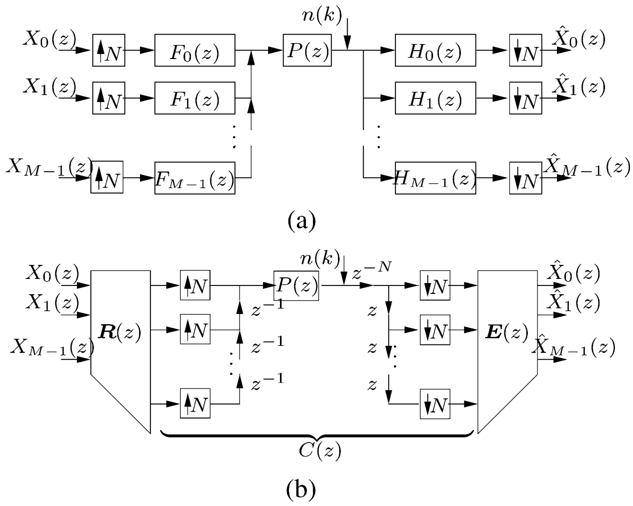

Fig. 1. (a) M-channel oversampled TMUX. (b) M-channel TMUX with reduced complexity.

Similarly, the causal synthesis bank of an oversampled M-band filter bank can defined as

$$\tilde{\boldsymbol{r}}(z) = [F_0(z) \;\; F_1(z) \;\; \cdots \;\; F_{M-1}(z)] = \boldsymbol{d}(z)\boldsymbol{R}(z^N) \tag{5}$$

where

$$\boldsymbol{R}(z^N) = \begin{bmatrix} R_{0,0}(z^N) & R_{0,1}(z^N) & \cdots & R_{0,M-1}(z^N) \\ R_{1,0}(z^N) & R_{1,1}(z^N) & \cdots & R_{1,M-1}(z^N) \\ \vdots & \vdots & \ddots & \vdots \\ R_{N-1,0}(z^N) & R_{N-1,1}(z^N) & \cdots & R_{N-1,M-1}(z^N) \end{bmatrix}. \tag{6}$$

Based on the transmultiplexers depicted in Fig. 1(a), the input/output relation is

$$\hat{\boldsymbol{X}}(z) = \mathcal{D}_N\left\{\tilde{\boldsymbol{e}}(z)z^{-1}P(z)\tilde{\boldsymbol{r}}(z)\right\}\boldsymbol{X}(z) \tag{7}$$

where $\mathcal{D}_N\{.\}$ is the downsampling-by-N operator, and z^{-1} is a delay inserted just before the analysis bank to synchronize the upsampling and downsampling stages of a given subchannel u, for $u = 0, \ldots, (N-1)$ [see the multirate structure in Fig. 1(b)]. By making use of (2) and (5), the input/output relation of the transmultiplexer reduces to

$$\hat{\boldsymbol{X}}(z) = \boldsymbol{E}(z)\boldsymbol{C}(z)\boldsymbol{R}(z)\boldsymbol{X}(z) \tag{8}$$

with

$$\boldsymbol{C}(z) = \mathcal{D}_N\left\{z^{-N+1}\boldsymbol{d}^T(z^{-1})z^{-1}P(z)\boldsymbol{d}(z)\right\}. \tag{9}$$

In Fig. 1(b), the additive noise at the receiver (analysis bank) is represented by $n(k)$.

Using the polyphase representation of $P(z)$ given by

$$P(z) = \sum_{n=0}^{N-1} P_n(z^N)z^{-n} \tag{10}$$

the channel matrix $\boldsymbol{C}(z)$ can be written as

$$\boldsymbol{C}(z) = z^{-1}\begin{bmatrix} P_0(z) & z^{-1}P_{N-1}(z) & \cdots & z^{-1}P_1(z) \\ P_1(z) & P_0(z) & \cdots & z^{-1}P_2(z) \\ \vdots & \vdots & \ddots & \vdots \\ P_{N-1}(z) & P_{N-2}(z) & \cdots & P_0(z) \end{bmatrix} \tag{11}$$

which is a pseudocirculant matrix. The nature of the channel matrix $\boldsymbol{C}(z)$ is described in the literature [6], [7]. In a PR system, the transceiver must be designed such that $\boldsymbol{E}(z)\boldsymbol{C}(z)\boldsymbol{R}(z) \equiv z^{-\Delta}\boldsymbol{I}$, where Δ is an integer, what is frequently achieved by inserting redundancy to the system, as detailed in the following section.

III. Paraunitary Redundant FIR Filter Bank

Using the filter bank and the channel matrix notations given in the previous section, one can describe a new transceiver and its design procedure, to generate a proper estimate $\boldsymbol{X}(z)$ of the input $\hat{\boldsymbol{X}}(z)$.

The conditions for zero-forcing (ZF) equalization for a redundant transmitter, irrespective of the channel zero locations, were reported in [3]. ZF is achieved by simply designing the equalizer in such a manner it inverts the frequency response of the channel. In fact, assuming a zeroth-order transmitter matrix, a Qth-order receiver, and a channel $P(z)$ with order N_c, the condition

$$Q \geq \left\lceil \frac{Nc}{N-M} \right\rceil - \left\lceil \frac{N_c}{N} \right\rceil + 1 \tag{12}$$

must be satisfied, where $\lceil x \rceil$ denotes the smallest integer greater than or equal to x.

The redundant memoryless SC transceiver pair has transfer matrices given by

$$\begin{aligned} \boldsymbol{R} &= \begin{bmatrix} \boldsymbol{I}_{M\times M} \\ \boldsymbol{0}_{L\times M} \end{bmatrix} \\ \boldsymbol{E} &= [\boldsymbol{G}_{M\times M} \;\; \boldsymbol{0}_{M\times L}] \end{aligned} \tag{13}$$

where $L = (N-M)$. This solution, frequently considered in the literature [2], [3], [7], introduces redundancy with zero padding, and by conveniently designing $\boldsymbol{G}$, (12) can be satisfied properly. Such system can be viewed as a time-domain multiplexer (TDM). For any Bth-order paraunitary transmitter/receiver pair, condition (12) states that a special $(Q+B)$th-order receiver, composed by the counterpart of the Bth-order transmitter and any Qth-order FIR matrix, is able to perfectly recover the transmitted information. Of course, for a perfectly equalized system, it is possible to design FIR precoder and decoder such that their product results in a delayed identity, as desired.

In this manner, an FIR transmitter is proposed, aiming to increase the immunity of the system to additive colored noise. Furthermore, the proposed transmitter satisfies paraunitariness, resulting in a simple receiver design and in a PR transmultiplexer. Such transmitter is given by

$$\boldsymbol{R}(z) = \begin{bmatrix} \boldsymbol{S}(z)_{M\times M} \\ \boldsymbol{0}_{L\times M} \end{bmatrix} \tag{14}$$

In this case, if $\boldsymbol{S}(z)$ is paraunitary, $\boldsymbol{R}(z)$ will also be. For a given FIR matrix $\boldsymbol{S}(z)$, paraunitariness holds if

$$\boldsymbol{S}(z)\tilde{\boldsymbol{S}}(z) = d\boldsymbol{I}, \quad \text{for any } z \tag{15}$$

where $\tilde{\boldsymbol{S}}(z) = \boldsymbol{S}^{T*}(1/z^*)$, d is a simple constant, and the superscript * represents the complex conjugate operation.

The entries of matrix $\boldsymbol{S}(z)$ can be composed by polynomials generated by lossless lattice structures [6], which are frequently used as part of the structure of modulated filter banks (MFBs) [6], leading to very simple transceivers, easily optimized to meet any optimality criterion. In this manner, one can design a paraunitary $\boldsymbol{R}(z)$. First, assume that $S_i(z)$, $i = 0, \ldots, (M-1)$, are such polynomials. Define

$$\boldsymbol{S}(z) = \boldsymbol{\Lambda}_0(z)\boldsymbol{A} + z^{-1}\boldsymbol{\Lambda}_1(z)\boldsymbol{B} \tag{16}$$

with

$$\begin{aligned}
\boldsymbol{\Lambda}_0(z) &= \mathrm{diag}\left\{S_0(-z^2), \ \ldots, \ S_{M-1}(-z^2)\right\} \\
\boldsymbol{\Lambda}_1(z) &= \mathrm{diag}\left\{S_{M-1}(-z^2), \ \ldots, \ S_0(-z^2)\right\} \\
\boldsymbol{A} &= \boldsymbol{I} - (-1)^K \boldsymbol{J} \\
\boldsymbol{B} &= -(-1)^K \boldsymbol{I} - \boldsymbol{J}
\end{aligned} \tag{17}$$

where $2K$ is the desired order of the matrix $\boldsymbol{S}(z)$. In the above formulation, $\boldsymbol{I}$ and $\boldsymbol{J}$ are, respectively, the identity matrix and the reversal identity matrix.

Using (14), (16), and (17) in (15), and evaluating z over the unity circle results in

$$\begin{aligned}
\boldsymbol{S}(z)\tilde{\boldsymbol{S}}(z) &= \boldsymbol{\Lambda}_0(z)\boldsymbol{A}^2\boldsymbol{\Lambda}_0(z^{-1}) + \boldsymbol{\Lambda}_1(z)\boldsymbol{B}^2\boldsymbol{\Lambda}_1(z^{-1}) \\
&= 2\left(\boldsymbol{\Lambda}_0(z)\boldsymbol{\Lambda}_0(z^{-1}) + \boldsymbol{\Lambda}_1(z)\boldsymbol{\Lambda}_1(z^{-1})\right)
\end{aligned} \tag{18}$$

since

$$\begin{aligned}
\boldsymbol{A}^2 &= 2\left(\boldsymbol{I} - (-1)^K\boldsymbol{J}\right) \\
\boldsymbol{B}^2 &= 2\left(\boldsymbol{I} + (-1)^K\boldsymbol{J}\right) \\
\boldsymbol{AB} &= \boldsymbol{BA} = \boldsymbol{0} \\
\boldsymbol{\Lambda}_0(z)\boldsymbol{J}\boldsymbol{\Lambda}_0(z^{-1}) &= \boldsymbol{\Lambda}_1(z)\boldsymbol{J}\boldsymbol{\Lambda}_1(z^{-1}).
\end{aligned} \tag{19}$$

The right-hand side of (18) can be written as

$$S_i(-z^2)S_i(-z^{-2}) + S_{M-1-i}(-z^2)S_{M-1-i}(-z^{-2}) = 1/2 \tag{20}$$

for $i = 0, \ldots, \lceil M/2 \rceil - 1$, which are the conditions for PR, and that can be met by making $S_i(z)$ and $S_{M-1-i}(z)$ as the upper and lower branch transfer functions of a two channel lossless lattice, for each i, as in [6]

$$\begin{bmatrix} S_i(z) \\ S_{M-1-i}(z) \end{bmatrix} = \prod_{k=1}^{K} \begin{bmatrix} \sin\theta_{ik} & z^{-1}\cos\theta_{ik} \\ \cos\theta_{ik} & -z^{-1}\sin\theta_{ik} \end{bmatrix} \begin{bmatrix} \sin\theta_{i0} \\ \cos\theta_{i0} \end{bmatrix}. \tag{21}$$

Since $\tilde{\boldsymbol{S}}(z)$ is non-causal, the receiver matrix can be designed as its delayed version, or simply

$$\boldsymbol{E}(z) = \left[z^{-1}\tilde{\boldsymbol{S}}(z) \quad \boldsymbol{0}_{M\times L}\right]. \tag{22}$$

A further step is to design $\boldsymbol{S}(z)$ efficiently in such a manner that the effects of the channel matrix can be minimized. An indirect approach is to minimize the stopband energy of $H_p(z)$, or

$$F_2 = \frac{1}{\pi}\int_{\omega_s}^{\pi} \left|H_p(e^{j\omega})\right|^2 d\omega \tag{23}$$

where

$$H_p(z) = \sum_{i=0}^{M-1} S_i(z^{2M})z^{-i} + z^{-2K+2}S_i(z^{-2M})z^{-2M+1-i}. \tag{24}$$

In this case, the polynomials $S_i(z)$, $i = 0, \ldots, M-1$, are treated as the $2M$ polyphase components of a linear-phase prototype filter $H_p(z)$, designed for M-channel PR modulated filter banks, such as cosine and sine-modulated filter banks (CMFBs and SMFBs), or DFT filter banks. The overlapping factor is $2K$. The prototype filter is optimized with the least-squares (LS) criterion in (23), by means of adjusting the angles θ_{ik} in (21). In this article, a quadratic programming (QP) routine was used to perform the optimization. For further information regarding the design of prototype filters for modulated filter banks, see [8] and references therein.

The minimization of F_2, after matrix multiplication by the modulation matrix $\boldsymbol{T}$, leads to a highly selective receiver, enabling efficient equalization in the frequency-domain, which will improve the performance of the system over non-flat channels with colored noise. Alternatively, the matrix $\boldsymbol{S}(z)$ can be optimized conveniently to reduce intersymbol and intercarrier interferences (ISI and ICI, respectively), for a given known channel matrix [9]. The overall distortion due to ISI and ICI is known as signal-to-interference ratio (SIR).

The proposed transceiver is closely related to the well known CMFB filter bank. In fact, if $L = 0$ and $\boldsymbol{C}_{IV}$ is the DCT type 4 matrix, then $\boldsymbol{C}_{IV}\tilde{\boldsymbol{S}}(z)$ is the CMFB analysis bank.

IV. Synchronization Delay and Post-Combiner Equalizer

This section presents MMSE equalizers to provide robustness against channel imperfections. The channel magnitude response may be nonflat, that is, $P(z) \neq z^{-\Delta}$, leading to high levels of ICI and ISI even if the transmultiplexer holds the PR property. The use of a synchronization delay and a post-combiner equalizer is required to reduce such imperfections [10]. The synchronization delay is placed immediately after the FIR channel $P(z)$ [see Fig. 1(a)], and maximizes, during the training period, the correlation of the received signal with a cN-sample delayed version of the corresponding training sequence, where c is a positive integer, and N is the downsampling/upsampling factor. As an alternative, such delay may be conveniently replaced by a channel shortening FIR filter. The solution for channel shortening is well known and will not be treated here [11].

The post-combiner equalizer is placed at the rightmost side of Fig. 1(b), and is composed by a unitary transform $\boldsymbol{T}$ and a tridiagonal matrix $\boldsymbol{W}(z)$ [10], with order Q, as illustrated in Fig. 2. The key point is that the unitary transform will decompose the received signal in M orthogonal bands, since $\boldsymbol{TE}(z)$ results in an MFB whose subband transfer functions are modu-

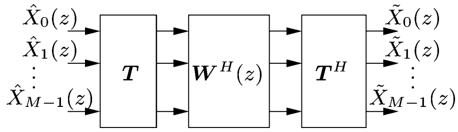

Fig. 2. M-channel post-combiner equalizer.

lated versions of the frequency selective prototype filter $H_p(z)$. Each band can be equalized considering only the immediately adjacent bands (upper and lower), reducing the computational complexity of the evaluation of the MMSE solution.

In the post-combiner equalizer of Fig. 2, define $\hat{\boldsymbol{W}}(z)$ as the MMSE solution when $\boldsymbol{T} = \boldsymbol{I}$. Then, for any orthonormal transform (including the identity matrix), one can write that

$$\boldsymbol{W}(z) = \boldsymbol{T}\hat{\boldsymbol{W}}(z)\boldsymbol{T}^H \tag{25}$$

where $(\cdot)^H$ is the conjugate transpose operation. The desired signal vector at the output of the equalizer and the non-transformed input signal vector are, respectively

$$\begin{aligned}\boldsymbol{x}(n) &= [x_0(n)\ x_1(n)\ \cdots\ x_{M-1}(n)]^T\\ \hat{\boldsymbol{x}}(n) &= [\hat{x}_0(n)\ \hat{x}_1(n)\ \cdots\ \hat{x}_{M-1}(n)]^T.\end{aligned} \tag{26}$$

If $\boldsymbol{T} = \boldsymbol{I}$, the MMSE solution can be evaluated with the aid of the following set of equations:

$$\begin{aligned}\bar{\boldsymbol{x}}(n) &= \left[\hat{\boldsymbol{x}}^T(n)\ \hat{\boldsymbol{x}}^T(n-1)\ \cdots\ \hat{\boldsymbol{x}}^T(n-Q+1)\right]^T\\ \boldsymbol{\mathcal{D}}(n_a,n_b) &= [\boldsymbol{x}(n_a)\ \cdots\ \boldsymbol{x}(n_b)]\\ \boldsymbol{\mathcal{X}}(n_a,n_b) &= [\bar{\boldsymbol{x}}(n_a)\ \cdots\ \bar{\boldsymbol{x}}(n_b)]\\ \boldsymbol{\mathcal{W}} &= \left[\boldsymbol{\mathcal{W}}_0^T\ \boldsymbol{\mathcal{W}}_1^T\ \cdots\ \boldsymbol{\mathcal{W}}_{Q-1}^T\right]^T\\ &= \left(\boldsymbol{\mathcal{X}}(n_a,n_b)\boldsymbol{\mathcal{X}}^H(n_a,n_b)\right)^{-1}\\ &\quad\times \boldsymbol{\mathcal{X}}(n_a,n_b)\boldsymbol{\mathcal{D}}^H(n_a,n_b)\end{aligned} \tag{27}$$

where Q is the order of the equalizer matrix $\boldsymbol{W}(z)$. The region of support in the time-domain used to estimate correlations lies in the range $[n_a, n_b]$. Since the equalizer includes only neighboring subchannels and using the fact that it is possible to decompose the equalizer matrix in the z-domain as a sum of constant matrices, one can write

$$\begin{aligned}\hat{\boldsymbol{W}}(z) &= \hat{\boldsymbol{W}}_0 + \hat{\boldsymbol{W}}_1 z^{-1} + \cdots + \hat{\boldsymbol{W}}_Q z^{-Q+1}\\ \hat{\boldsymbol{W}}_q &= \text{tridiag}\{\boldsymbol{\mathcal{W}}_q\} \qquad \text{for } q = 0,\ldots,Q-1\end{aligned} \tag{28}$$

where $\text{tridiag}\{\boldsymbol{\mathcal{W}}\}$ represents a tridiagonal matrix composed by the lower adjacent diagonal, the main diagonal, and the upper adjacent diagonal of matrix $\boldsymbol{\mathcal{W}}$.

In this article, the orthonormal transformation $\boldsymbol{T}$ was the $\boldsymbol{C}_{IV}$ matrix whenever the input signals were real, or the DFT matrix if the signals under consideration were complex.

For comparison purposes, the computational complexity of the proposed redundant FIR (RFIR) structure, the constant matrix redundant (RC) transmitter (SC), and the OFDM system, measured by the number of multiplications per output sample $(\mathcal{C})$, is given in Table I. In this table, Tx and Rx are the transmitter and receiver, and M and $2K$ are the number of bands and the order of each bank of the RFIR transceiver, respectively.

TABLE I
MULTIPLICATIONS PER OUTPUT SAMPLE $(\mathcal{C})$

	Tx	Rx
RFIR	$2K$	$3(Q+1)+2K$
RC	–	$3(Q+1)$
OFDM	$\log_2 M$	$3(Q+1)+\log_2 M$

The complexity of the receiver is higher due to equalization (synchronization delay or channel shortening complexities are usually negligible on those systems). A suitable application of the proposed system is mobile communication, requiring lower complexity in the transmitter in the uplink (mobile to base station).

V. SIMULATION EXAMPLES

Example 1: This example compares the SC system designed in [3] with the RFIR filter bank proposed in this work, and the traditional OFDM system. The configuration parameters are $M = 12$, and $L = 4$ ($N = 16$). For the BER evaluation, a total run of 100 Monte Carlo iterations were executed, with data information represented by 6000 BPSK symbols for each subband. The channel is given by the first 30 samples of the impulse response [3] $P(z) = 0.314(1 + 0.067z^{-1} + 0.394z^{-2} + 0.784z^{-3}/1 - 1.084z^{-1} + 0.946z^{-2} - 0.157z^{-3})$, where it is clear that the redundancy inserted is not enough to reduce the channel matrix $\boldsymbol{C}(z)$ into a constant. The simulations were carried out by adding white noise, or white plus colored noise. The colored noise had variance $\sigma_c^2 = 10\sigma_y^2$, where σ_y^2 was the variance of the noiseless transmitted signal. The channel was normalized by the L_2 norm. Fig. 3(a) and (b) shows the magnitude response of the channel and the power spectrum of the colored noise, respectively. The synchronization delay was estimated with 20 consecutive training blocks of length N, whereas the post-combiner used a region of support of $n_b - n_a = 300$ blocks. Such parameters were chosen to guarantee a satisfactory estimate of signal correlations. The order of the post-combiner was chosen as $Q = 4$, which leads to an appropriate equalization of the channel under consideration [3]. Despite the fact that the choice for the order does not satisfy (12), ZF is not under consideration and is far from a necessary condition. The receiver transformation matrix was $\boldsymbol{T} = \boldsymbol{C}_{IV}$. The transceiver proposed in this work is called RFIR, whereas *RC* is the redundant transmitter with constant matrix (13) presented in [3]. The RFIR was designed by minimizing F_2 in (23) independent on the channel impulse response. The overlapping factor used was $2K = 8$, which guaranteed reasonable performance and low computational complexity.

The BER as a function of the SNR is shown in Fig. 3(c), where it is clear that for a frequency selective channel with additive white noise, the performance of both RFIR and *RC* systems was the same. However, in the presence of additive colored noise the RFIR outperformed the *RC* by 5 dB for $\text{BER} = 10^{-2}$ due to the high selectivity of the receiver/equalizer pair, enabling efficient equalization in the frequency-domain. The orthogonal frequency division multiplexing (OFDM) system performed poorly because the channel $P(z)$ under consideration

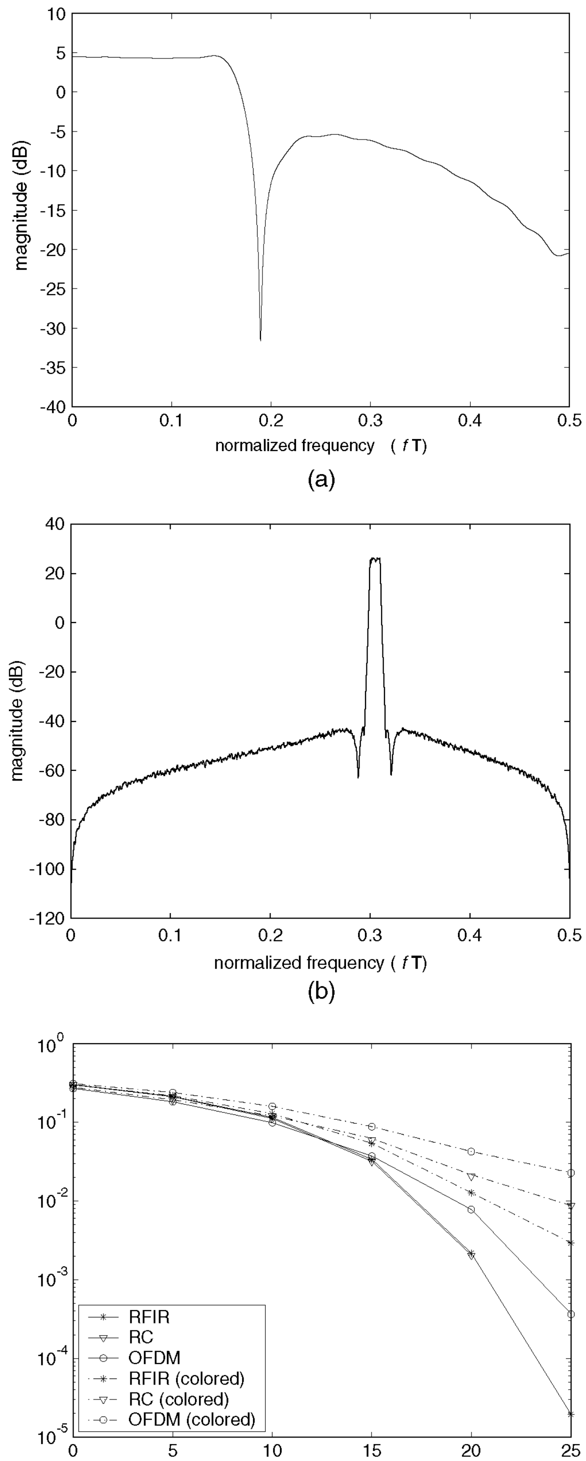

Fig. 3. (a) Magnitude response of channel in Example 1. (b) Power spectrum of colored noise. (c) BER as a function of the SNR for different channel impairments in Example 1.

degraded severely one subchannel of the transceiver, imposing strong attenuation around the normalized frequency $f\mathrm{T} \approx 0.2$. FDMs like OFDM are usually more susceptible to frequency selective channels than TDMs [2], [4]. The TDMs are represented by the proposed RFIR and the *RC* in this article.

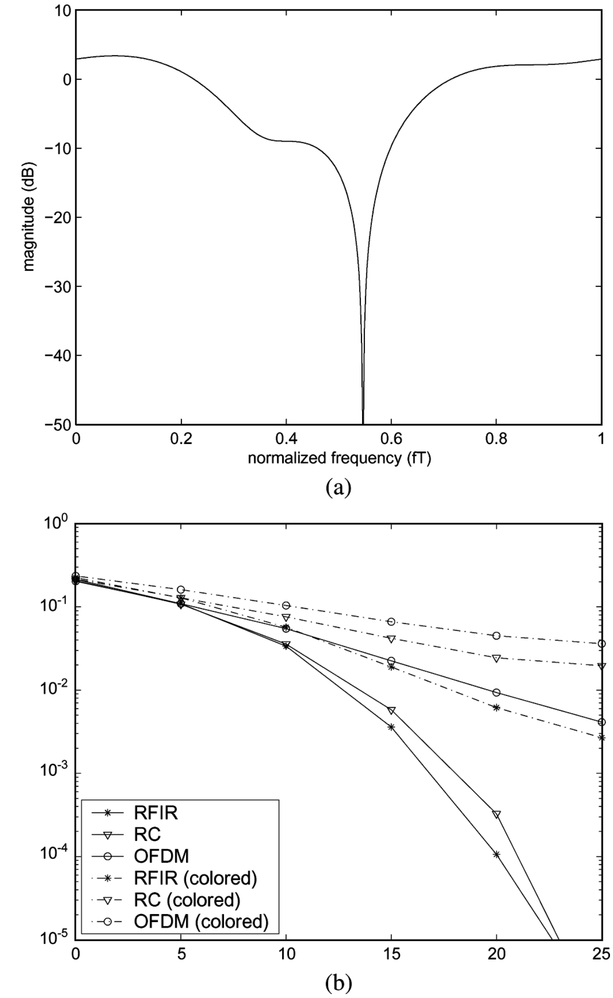

Fig. 4. Example 2. (a) Magnitude response of the channel. (b) BER as a function of the SNR for different channel impairments.

The computational complexity of the structures under considerantion are: $\mathcal{C}_{\mathrm{RFIR}} = 16$, $\mathcal{C}_{\mathrm{RC}} = 0$, and $\mathcal{C}_{\mathrm{OFDM}} \approx 4$ in the transmitter; $\mathcal{C}_{\mathrm{RFIR}} = 31$, $\mathcal{C}_{\mathrm{RC}} = 15$, and $\mathcal{C}_{\mathrm{OFDM}} \approx 19$ in the receiver. The increased complexity of the RFIR is compensated by its improved performance against OFDM in all scenarios, and over the *RC* for colored noise.

Example 2: A comparison was performed with the SC system studied in [2] and the OFDM system, with $M = 64$ and $L = 3$ and a QPSK constellation given by symbols $\pm 1 \pm j1$. The channel is given by $P(z) = -0.3699 - j0.5782 - (0.4053 + j0.5750)z^{-1} - (0.0834 + j0.0406)z^{-2} + (0.1587 - j0.0156)z^{-3}$, as used in the second example of [2]. The magnitude response of the channel is depicted in Fig. 4(a). The colored noise used was the same as in Example 1. The RFIR was designed as described in Example 1, and the overlapping factor chosen was $2K = 16$, providing satisfactory trade-off between BER performance and computational complexity. The post-combiner equalizer used as transform is the DFT matrix, that is, $\boldsymbol{T} = \boldsymbol{W}_M$, and had order $Q = 0$. The BER is depicted in Fig. 4(b), which confirms the improved performance of the proposed structure over the

SC transmitter, for colored noise, and over the OFDM system in all scenarios.

In this case, the computational complexities are: $\mathcal{C}_{\text{RFIR}} = 16$, $\mathcal{C}_{\text{RC}} = 0$, and $\mathcal{C}_{\text{OFDM}} = 8$ in the transmitter; $\mathcal{C}_{\text{RFIR}} = 19$, $\mathcal{C}_{\text{RC}} = 3$, and $\mathcal{C}_{\text{OFDM}} = 11$ in the receiver. In this example, the complexities of both RFIR and OFDM are very close, but the RFIR still performed significantly better than OFDM. Again, the SC system had negligible complexity, but worse performance for colored noise than the RFIR.

VI. Conclusion

This work presented an efficient redundant paraunitary FIR transceiver with improved performance over OFDM and traditional single carrier transmitters, in terms of achievable BER in the presence of additive colored noise. The improvement is mostly due to the memory inserted in the transceiver structure, leading to a highly selective receiver/equalizer pair, after the appropriate optimization of the structure. An LS optimization technique was proposed to provide the desired spectral containment. The overall computational complexity for the proposed structure is comparable to CMFB filter banks.

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